สัญญาเลขที่ MRG5180225

รายงานวิจัยฉบับสมบูรณ์

โครงการ: การศึกษาจักรวาลวิทยาโดยการประยุกต์ใช้แบบจำลองจากทฤษฎีสตริง
Study Cosmology via String Theory Inspired Models

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สนับสนุนโดยสำนักงานคณะกรรมการการอุดมศึกษา และสำนักงานกองทุนสนับสนุนการวิจัย
(ความเห็นในรายงานนี้เป็นของผู้วิจัย สกอ. และ สกว. ไม่จำเป็นต้องเห็นด้วยเสมอไป)

Abstract (บทคัดย่อ)

Project Code : MRG5180225
(รหัสโครงการ)

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(ชื่อโครงการ)

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(ระยะเวลาโครงการ)

โครงการวิจัยนี้มีจุดมุ่งหมายในการศึกษาเสถียรภาพของมิติพิเศษ หรือ Extra-dimension ซึ่งม้วนตัวอยู่ และ วิเคราะห์ให้เห็นถึงความสัมพันธ์ของมิติพิเศษเหล่านี้กับการขยายตัวด้วยความเร่งของเอกภพ โดยในแบบจำลองที่ได้ทำการศึกษานั้นการขยายตัวด้วยความเร่งเกิดขึ้นอย่างเป็นธรรมชาติเมื่อขนาดมิติพิเศษมีเสถียรภาพแล้ว และ Casimir energy ของสนามในมิติพิเศษจะทำหน้าที่เป็นค่าคงที่จักรวาลในเอกภพ 4 มิติของเรา

ผู้วิจัยทำการปรับปรุงกลไกการสร้างเสถียรภาพของมิติพิเศษใน แบบจำลองก่อนหน้านี้ โดยการพิจารณาบทบาทของสนามเวกเตอร์ที่มีขนาดคงที่หรือที่เรียกว่า aether field ซึ่งมีทิศทางอยู่ในมิติพิเศษ ผู้ทำการวิจัยได้ค้นพบว่า aether field มีผลทำให้อิทธิพลของ gradient ของศักย์ยังผลรวมมีค่าน้อยลง ซึ่งจะช่วยให้มิติพิเศษสามารถเข้าสู่เสถียรภาพได้

Keywords : Cosmology, Dark Energy, String Theory, Extra-dimension

Executive Summary

โครงการวิจัยนี้มีจุดมุ่งหมายในการศึกษาเสถียรภาพของมิติพิเศษ หรือ Extra-dimension ซึ่งม้วนตัวอยู่ และ วิเคราะห์ให้เห็นถึงความสัมพันธ์ของมิติพิเศษเหล่านี้กับการขยายตัวด้วยความเร่งของเอกภพ โดยในแบบจำลองที่ได้ทำการศึกษานั้นการขยายตัวด้วยความเร่งเกิดขึ้นอย่างเป็นธรรมชาติเมื่อขนาดมิติพิเศษมีเสถียรภาพแล้ว และ Casimir energy ของสนามในมิติพิเศษจะทำหน้าที่เป็นค่าคงที่จักรวาลในเอกภพ 4 มิติของเรา

งานวิจัยอาจแบ่งออกได้เป็นสองส่วนหลักๆ ในส่วนแรกนั้นผู้วิจัยได้ทำการปรับปรุงแบบจำลองของ B.R. Greene และ J. Levin โดยการพิจารณา Volume และ Shape moduli ของ extra dimension ที่มี topology เป็น Torus 2 มิติ ซึ่งในทางคณิตศาสตร์สมบัติของ space นี้สามารถอธิบายด้วย moduli fields τ_1 และ τ_2 ซึ่งทำหน้าที่อธิบายรูปทรงหรือ shape ของมิติพิเศษ โดยเราได้พบว่าค่าต่ำสุดของ Casimir energy จะอยู่ที่จุด $\tau_1 = \pm 1/2$ และ $\tau_2 = \sqrt{3}/2$ บน Moduli space และค่าต่ำสุดที่เคยเสนอไว้โดย B.R. Greene และ J. Levin ซึ่งอยู่ที่ตำแหน่ง $\tau_1 = 0$, $\tau_2 = 1$ ของ Moduli space แท้จริงแล้วเป็นเพียงจุด saddle point จากการวิเคราะห์ทางพลศาสตร์ของระบบพบว่าการรบกวนเพียงเล็กน้อยที่บริเวณจุด saddle point จะทำให้สภาวะของเอกภพเคลื่อนตัวลงสู่จุดที่มีพลังงานต่ำสุด ณ ตำแหน่งค่าต่ำสุดที่ค้นพบใหม่และเข้าสู่เสถียรภาพในที่สุด สิ่งที่น่าสนใจในแบบจำลองลักษณะนี้ก็คือ ค่าคงที่จักรวาลที่ปรากฏในอวกาศ 4 มิติจะขึ้นอยู่กับรัศมีการม้วนตัวของมิติพิเศษ นอกจากนี้ยังได้ทำการคำนวณ shear viscosity ในมิติพิเศษโดยพบว่าจะมีค่าขึ้นอยู่กับ Hubble constant (หรืออัตราการขยาย) ของอวกาศ 4 มิติที่ ณ ตำแหน่งที่มิติพิเศษเริ่มมีเสถียรภาพ

สำหรับในส่วนที่สอง ผู้วิจัยได้ทำการปรับปรุงกลไกการสร้างเสถียรภาพของมิติพิเศษในแบบจำลองก่อนหน้านี้ โดยการพิจารณาบทบาทของ การสลายสมมาตรแบบ Lorentz ในมิติพิเศษ โดยสนามเวกเตอร์ที่มีขนาดคงที่หรือที่เรียกว่า aether field ซึ่งมีทิศทางในมิติพิเศษ ในแบบจำลองเดิมที่เสนอขึ้นโดย B.R. Greene และ J. Levin นั้นศักย์ยังผลเนื่องจาก Casimir energy จะถูกกลบโดยอิทธิพลของสสารในเอกภพ (อันได้แก่ สสารมืด หรือ Dark matter ที่มีสมบัติเป็น non-relativistic matter) อันจะทำให้มิติพิเศษไม่สามารถเข้าสู่เสถียรภาพได้ ผู้ทำการวิจัยได้ค้นพบว่า aether field ที่มีทิศทางชี้ไปในมิติพิเศษมีผลทำให้อิทธิพลของ gradient ของศักย์ยังผลรวมมีค่าน้อยลง ซึ่งเป็นสิ่งที่ยังไม่เคยมีการค้นพบมาก่อน และ ข้อเท็จจริงนี้มีส่วนช่วยให้มิติพิเศษสามารถเข้าสู่เสถียรภาพได้ การค้นพบนี้ถือว่าเป็นสิ่งที่น่าสนใจ และอยู่นอกเหนือการคาดหมายจากแผนการเริ่มต้นของโครงการ ซึ่งจะนำไปประยุกต์ใช้กับกระบวนการอื่นๆ ที่เกี่ยวข้องกับวิวัฒนาการของเอกภพได้ และซึ่งผู้วิจัยวางแผนที่มีการศึกษาค้นคว้าโดยละเอียดต่อไปในอนาคต

งานวิจัยนี้ได้ตีพิมพ์ลงในวารสาร Journal of High-Energy Physics จำนวนสองฉบับ ดังแสดงไว้ในภาคผนวก

เนื้อหาของโครงการ

1) วัตถุประสงค์ของโครงการ

(1.1) เพื่อศึกษาผลของ Casimir energy ใน extra dimensions กับการขยายตัวด้วยความเร่งของเอกภพ

(1.2) เพื่อวิเคราะห์เสถียรภาพของ extra dimension ในกรณีที่มี Casimir energy ในมิติเหล่านั้น โดยการพิจารณาปริภูมิของพารามิเตอร์ที่เกี่ยวข้อง (moduli space) โดยเฉพาะอย่างยิ่ง volume stabilization และ shape stabilization ของ extra dimensions

(1.3) เพื่อศึกษาความเป็นไปได้ที่ Casimir energy ใน extra dimension จะมีอันตรกริยากับสสารอื่นในเอกภพ เช่น มวลสารมืด (dark matter) อันจะส่งผลกับวิวัฒนาการของเอกภพเอง

2) วิธีทดลอง

(2.1) ศึกษาแบบจำลองของเอกภพซึ่งได้รับแรงบันดาลใจมาจากทฤษฎีสตริง ซึ่งเสนอโดย B.R. Greene และ J. Levin โดยที่แบบจำลองดังกล่าวสมมุติว่าเอกภพมีจำนวนมิติมากกว่า 4 มิติ และมิติพิเศษที่เพิ่มขึ้นนี้ขดตัวอยู่โดยมีลักษณะทาง topology เป็น Torus ที่มีรัศมีขนาดเล็ก โดยที่ผลของ Casimir energy ซึ่งเป็น vacuum energy ของสนามในextra dimension จะทำหน้าที่เป็น dark energy ในเอกภพ 4 มิติที่เราอาศัยอยู่ และการขยายตัวด้วยความเร่งของมิติปกติ จะสอดคล้องกับเสถียรภาพของ extra dimensions

(2.2) เนื่องจากงานวิจัยของ B.R. Greene และ J. Levin ยังมิได้พิจารณาเสถียรภาพของ extra dimension ให้ครอบคลุมปริมิภูมิของพารามิเตอร์ที่อธิบายพลวัตของ extra dimension ซึ่งเราเรียกปริภูมิดังกล่าวว่า Moduli space ซึ่งในโครงการนี้จะทำการปรับปรุงแบบจำลองของ B.R. Greene และ J. Levin โดยการพิจารณา Volume และ Shape moduli ของ extra dimension ที่มี topology เป็น Torus 2 มิติ

(2.3) ศึกษา Casimir energy ของระบบสนามซึ่งถูกขังอยู่ในอวกาศที่มีลักษณะเป็น Torus ซึ่งเป็น genus 1 surface และพิจารณาผลของพลังงานนี้ กับการขยายตัวของเอกภพ และวิเคราะห์เสถียรภาพของ extra dimensions ด้วยวิธีการทางตัวเลขโดยใช้โปรแกรม Mathematica เข้าช่วย

(2.4) ศึกษาความเป็นไปได้ที่ Casimir energy ใน extra dimensions ในแบบจำลองนี้จะส่งผลกับวิวัฒนาการของเอกภพ ผ่านทางปรากฏการณ์อื่นๆ เช่น อันตรกิริยาระหว่างมวลสารในเอกภพกับมวลสารในมิติพิเศษเป็นต้น

(2.5) ศึกษาความเป็นไปได้ในการประยุกต์ใช้ผลที่ได้จากการศึกษาในข้อ 1-4 เพื่อนำมาสร้างกลไกการพองตัว (Inflation) ของเอกภพในภาวะแรกเริ่ม (Early Universe)

(2.6) ศึกษาผลของ Lorentz violating field หรือ aether field กับวิวัฒนาการของเอกภพโดยเฉพาะอย่างยิ่งผลของสนามนี้ต่อเสถียรภาพของมิติพิเศษ

(2.7) ติดต่อสำนักพิมพ์เพื่อตีพิมพ์เผยแพร่ผลงานวิจัยในวารสาร Journal of High Energy Physics

3) ผลการทดลอง

(3.1) พิสูจน์ว่า Volume และ Shape moduli ของ extra dimension ที่มี topology เป็น Torus 2 มิติซึ่งในทางคณิตศาสตร์สมบัติของ space นี้สามารถอธิบายด้วย moduli fields τ_1 และ τ_2 ซึ่งทำหน้าที่อธิบาย รูปทรงหรือ shape ของมิติพิเศษนั้น จะมีค่าต่ำสุดของ Casimir energy จะอยู่ที่จุด $\tau_1 = \pm 1/2$ และ $\tau_2 = \sqrt{3}/2$ บน Moduli space และค่าต่ำสุดที่เคยเสนอไว้โดย B.R. Greene และ J. Levin ซึ่งอยู่ที่ตำแหน่ง $\tau_1 = 0$, $\tau_2 = 1$ ของ Moduli space แท้จริงแล้วเป็นเพียงจุด saddle point

(3.2) ค้นพบว่า aether field ที่มีทิศทางชี้ไปในมิติพิเศษมีผลทำให้อิทธิพลของ gradient ของศักย์ยังผลรวม มีค่าน้อยลง ซึ่งเป็นสมบัติที่ยังไม่เคยมีการค้นพบมาก่อน และ ข้อเท็จจริงนี้มีส่วนช่วยให้มิติพิเศษสามารถเข้าสู่เสถียรภาพได้ แม้ในกรณีที่มี non-relativistic matter อยู่ในเอกภพ

4) สรุปและวิจารณ์ผลการทดลอง และข้อเสนอแนะสำหรับงานวิจัยในอนาคต

ผลการวิเคราะห์ความเป็นไปได้ในการนำแบบจำลองนี้ไปอธิบายปรากฏการณ์การพองตัว (Inflation) ของเอกภพในยุคแรกเริ่มนั้นมีแนวโน้มว่า ลำพังศักย์ยังผลจากพลังงาน Casimir เพียงอย่างเดียว อาจไม่เพียงพอที่จะทำให้เกิดการพองตัวที่มากพอและสอดคล้องกับข้อมูลจาก Cosmic Microwave Background Radiation (CMB) อย่างไรก็ตาม ผลจาก aether field ที่สามารถลดอิทธิผลของ potential gradient อาจจะทำให้ moduli field มีพลวัตต์ที่เหมาะสมและสามารถประพฤติตัวเป็น Inflaton field เพื่อทำให้เกิดการพองตัวในลักษณะที่ต้องการได้ ซึ่งจะเป็นประเด็นที่ต้องทำการศึกษาต่อไป

5) Output จากโครงการวิจัยที่ได้รับทุนจาก สกว.

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ (ระบุชื่อผู้แต่ง ชื่อเรื่อง ชื่อวารสาร ปี เล่มที่ เลขที่ และหน้า) หรือผลงานตามที่คาดไว้ในสัญญาโครงการ

   1.1) ชื่อผู้แต่ง Burikham P., **Chatrabhuti A.**, Patcharamaneepakorn P. Pimsamarn K. ชื่อเรื่อง *Dark energy and Moduli stabilization of extra dimension in* $M^{(1+3)}xT^2$ *spacetime* วารสาร **Journal of High-Energy Physics** เล่มที่ 07 เลขที่ 013 ปี 2008
   1.2) ชื่อผู้แต่ง **Chatrabhuti A.**, Patcharamaneepakorn P., Wongjun P. ชื่อเรื่อง *Aether field, Casimir energy and stabilization of the extra dimension*, วารสาร **Journal of High-Energy Physics** เล่มที่ 08 เลขที่ 019 ปีที่ 2009

2. การนำผลงานวิจัยไปใช้ประโยชน์
   - เชิงพาณิชย์ (มีการนำไปผลิต/ขาย/ก่อให้เกิดรายได้ หรือมีการนำไปประยุกต์ใช้โดยภาคธุรกิจ/บุคคลทั่วไป)

     ไม่มี
   - เชิงนโยบาย (มีการกำหนดนโยบายอิงงานวิจัย/ เกิดมาตรการใหม่/ เปลี่ยนแปลงระเบียบข้อบังคับหรือวิธีทำงาน)

     ไม่มี
   - เชิงสาธารณะ (มีเครือข่ายความร่วมมือ/สร้างกระแสความสนใจในวงกว้าง)

     ไม่มี
   - เชิงวิชาการ (มีการพัฒนาการเรียนการสอน/สร้างนักวิจัยใหม่)

     มีนิสิตปริญญาเอกร่วมในงานวิจัย คือ นายพิทยุทธ วงศ์จันทร์

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Dark energy and moduli stabilization of extra dimensions in $\mathbb{M}^{1+3} \times \mathbb{T}^2$ spacetime

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ABSTRACT: Recently, it was found by Greene and Levin that the Casimir energy of certain combinations of massless and massive fields in space with extra dimensions play a crucial role in the accelerated expansion of the late-time universe and therefore it could serve as a candidate for the dark energy. It also provides a mechanism in stabilizing the volume moduli of extra dimensions. However, the shape moduli of the extra dimensions were never taken into account in the previous work. We therefore study the stabilization mechanism for both volume and shape moduli due to the Casimir energy in $\mathbb{M}^{1+3} \times \mathbb{T}^2$. The result of our study shows that the previously known local minimum is a saddle point. It is unstable to the perturbations in the direction of the shape moduli. The new stable local minima stabilizes all the moduli and drives the accelerating expansion of the universe. The cosmological dynamics both in the bulk and the radion pictures are derived and simulated. The equations of state for the Casimir energy in a general torus are derived. Shear viscosity in extra dimensions induced by the Casimir density in the late times is identified and calculated, it is found to be proportional to the Hubble constant.

KEYWORDS: Large Extra Dimensions, Cosmology of Theories beyond the SM.

Contents

1. Introduction

According to the latest data on Type Ia Supernovae [1] and Cosmic Microwave Background Radiation (CMBR) [2], it is strongly believed that the universe consists of a sort of vacuum energy, namely dark energy, which contributes the accelerated expansion in three-dimensional space. Unfortunately, the exact form of the dark energy has not yet been uncovered until now. The prominent candidates for dark energy are the cosmological constant, and models of scalar fields, such as the quintessence and moduli fields.

In the standard cosmological model where the acceleration of the universe is taken into account by a positive cosmological constant term, dark energy contributes largely, more than 70 % of the total density of the universe [2]. This number (roughly $10^{-11}\,\mathrm{eV}^4$) seems arbitrarily small and the known mechanisms, such as the popular TeV-scale supersymmetry (SUSY) breaking scenario or any top-down high-scale particle physics mechanisms, fail to produce it.

In recent years, theories with large extra dimensions have received an explosion of interests as they provide new solution to the hierarchy problem. Recently, it was found that Casimir energy of massless and massive fields embedded in higher-dimensional spacetime could play a crucial role of dark energy with additional significant properties [3, 4].

The Casimir energy not only drives the expansion of universe acceleratedly, but also stabilizes the volume moduli of extra dimensions. However, the shape moduli, τ_1, τ_2, were not included in the work of Greene and Levin. In this work we therefore take into account these moduli in the cosmological dynamics by assuming that the extra dimensions are $\mathbb{T}^2$. The phenomenological implications of nontrivial shape moduli were pointed out in [5 – 7]. Shape moduli can have dramatic effects on the Kaluza-Klein spectrum, for example, they can induce level-crossings and varying mass gaps. They can also help to eliminate light KK states. It should be interesting to investigate the role of shape moduli in cosmology.

Our work employed the calculation of Casimir energy in the non-trivial space $\mathbb{M}^4 \times \mathbb{T}^2$. The Casimir energy is the vacuum energy contributed from the quantum fluctuation of fields which satisfy certain boundary conditions. In fact, the Casimir energy in various spaces including a distorted torus was studied in earlier works [8 – 10, 4]. The standard approach for determining the Casimir energy is the zeta function regularization [11].

Our result shows that the minimum of potential in the previous work [3] ($\tau_1 = 0, \tau_2 = 1$) was the unstable local minimum while the true local minimum locates at specific points in the moduli space, $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$, confirming the result of ref. [4]. At this local minimum the potential stabilizes all moduli and also sources the accelerated expansion of the four dimensional universe.

This paper is organized as follows. In section 2 we review cosmological dynamics on $\mathbb{M}^{1+n} \times \mathbb{T}^p$ spacetime. In section 3 we present the mathematical calculation to determine the Casimir energy of massive and massless fields in the spacetime with toroidally compactified extra dimensions. Then we go on to construct effective potential contributed by Casimir energy of massive and massless field in $\mathbb{M}^{1+3} \times \mathbb{T}^2$ spacetime in section 4. The numerical evidences of the stability of moduli space are presented in section 5. In section 6 we present our conclusions.

2. Cosmological dynamics in $\mathbb{M}^{1+n} \times \mathbb{T}^p$

Our study of cosmological dynamics is based upon the application of Einstein's general relativity on the product space $\mathbb{M}^{1+n} \times \mathbb{T}^p$, between a $(1 + n)$-dimensional spacetime and a p-dimensional toroidally-compactified space. As a whole, the total number of spatial dimensions is $d = n + p$. We assume the cosmological ansatz

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + h_{ij}(x)dy^i dy^j, \tag{2.1}$$

where the metric h_{ij} represent the p-dimensional compact space with $i, j = 1, \ldots, p$ and $g_{\mu\nu}$ for the $(1 + n)$-dimensional noncompact spacetime with $\mu, \nu = 0, \ldots, n$. Let's assume also that the metric only depends on the noncompact coordinates x^μ. The compact coordinates are $0 \leq y^i \leq 2\pi$.

In this paper, we focus our effort on the cosmological dynamics of a 4-dimensional spacetime with two extra dimensions ($n = 3$ and $p = 2$). The metric of two-dimensional torus $\mathbb{T}^2$ takes the form

$$(h_{ij}) = \frac{b^2}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}, \tag{2.2}$$

where $\tau = \tau_1 + i\tau_2$ is the complex structure (or shape moduli) and b^2 is the Kähler structure (or volume moduli). In cosmology, it is customary to write $g_{\mu\nu} = a^2(t)\eta_{\mu\nu}$ and $h_{ij} = h_{ij}(t)$. In the next sections, we will assume that Casimir energy in compact direction, $\rho_{(d+1)D}$, plays the roles of the dominant energy content in the universe. By using Einstein equations in (1 + 5)-dimensional spacetime, we obtain the following equations governing the cosmological dynamics:

$$3H_a^2+H_b^2+6H_aH_b-\frac{1}{4\tau_2^2}(\dot{\tau}_1^2+\dot{\tau}_2^2) = 8\pi G\rho_{6D}, \tag{2.3}$$

$$\dot{H}_a + 3H_a^2 + 2H_aH_b = \frac{8\pi G}{4}\left\{2\rho_{6D}+\left[1-\left(\frac{\tau_1}{\tau_2}\right)^2\right] b\partial_b\rho_{6D} -2\tau_1\partial_{\tau_1}\rho_{6D}+\frac{2\tau_1^2}{\tau_2}\partial_{\tau_2}\rho_{6D}\right\}, \tag{2.4}$$

$$\dot{H}_b+2H_b^2+3H_aH_b = -\frac{8\pi G}{4}\left\{-2\rho_{6D}+\left[1-\left(\frac{\tau_1}{\tau_2}\right)^2\right] b\partial_b\rho_{6D} -2\tau_1\partial_{\tau_1}\rho_{6D}+\frac{2\tau_1^2}{\tau_2}\partial_{\tau_2}\rho_{6D}\right\}, \tag{2.5}$$

$$\ddot{\tau}_1 + \left(3H_a + 2H_b - 2\frac{\dot{\tau}_2}{\tau_2}\right)\dot{\tau}_1 = -16\pi G\tau_2^2\left\{\frac{b\tau_1}{2\tau_2^2}\partial_b\rho_{6D}+2\partial_{\tau_1}\rho_{6D}-\frac{\tau_1}{\tau_2}\partial_{\tau_2}\rho_{6D}\right\}, \tag{2.6}$$

$$\frac{\ddot{\tau}_2}{\tau_2}+\frac{\dot{\tau}_1^2-\dot{\tau}_2^2}{\tau_2^2}+3H_a\frac{\dot{b}}{\tau_2}+2H_b\frac{\dot{\tau}_2}{\tau_2} = 8\pi G\left\{\frac{b\tau_1^2}{\tau_2^2}\partial_b\rho_{6D}+2\tau_1\partial_{\tau_1}\rho_{6D} -2\tau_2\left[1+\left(\frac{\tau_1}{\tau_2}\right)^2\right]\partial_{\tau_2}\rho_{6D}\right\}. \tag{2.7}$$

where G is the 6-D gravitational constant. We have defined the Hubble constants $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$, where a dotted quantity represents the corresponding time derivative and ρ_{6D} is the casimir energy density in six dimensional spacetime.

2.1 Dynamics in the radion picture

Equations of motion (2.3)–(2.7) can be obtained by varying the $d+1$-dimensional Einstein-Hilbert action:

$$S = \int d^{1+n}xd^py\sqrt{-gh}\left\{\frac{M_*^{d-1}}{16\pi}\mathcal{R}_{(1+d)} - \rho_{(1+d)D}(h^{ij})\right\}, \tag{2.8}$$

with $n = 3$ and $p = 2$, where $\rho_{(1+d)D}(h^{ij})$, $\mathcal{R}_{(1+d)}$ and M_* are the Casimir energy density, Ricci scalar and the Planck mass in $(1+d)$-dimensional spacetime respectively. For later purpose, it is useful to perform KK-dimensional reduction of the above action from $(1+d)$ to $(1+n)$-dimensional spacetime and Weyl rescaling $g_{\mu\nu E} = \Omega^{\frac{2}{n-1}}g_{\mu\nu}$; $\Omega = M_*^{d-1}V_p/m_{pl}^{n-1}$, the action takes the form

$$S=\int d^{1+n}x\sqrt{-g_E}\left\{\frac{m_{pl}^{n-1}}{16\pi}\left[\mathcal{R}_E+g_E^{\mu\nu}\left(\frac{1}{1-n}\nabla_\mu \ln\sqrt{h}\nabla_\nu \ln\sqrt{h}+\frac{1}{4}\nabla_\mu h^{ij}\nabla_\nu h_{ij}\right)\right]-U(h^{ij})\right\}. \tag{2.9}$$

Note that the subscript E denotes the Einstein frame variables. Here, $V_p = \int d^p y \sqrt{h} = (2\pi b)^p \equiv l^p$ is the (invariant) volume of extra dimensions, m_{pl} and $\mathrm{U}(h^{ij}) = \Omega^{\frac{1+n}{1-n}} V_p \rho_{(1+d)D}(h^{ij}) = \Omega^{\frac{1+n}{1-n}} \rho_{(1+n)D}(h^{ij})$ are the Planck mass and the effective potential in $1+n$-dimensional spacetime respectively. We can also take $\rho_{(1+n)D}(h^{ij})$ to be the Casimir energy density in $(1+n)$-dimensional spacetime.

Since we are interested in the $n=3$, $p=2$ case, by using the metric of two-dimensional torus defined in eq. (2.2), the action in eq. (2.9) can be written as

$$S=\int d^4x\sqrt{-g_E}\left\{\frac{m_{pl}^2}{16\pi}\left[\mathcal{R}_E - \frac{1}{2}g_E^{\mu\nu}(\nabla_\mu\psi\nabla_\nu\psi + e^{-2\phi_2}\nabla_\mu\phi_1\nabla_\nu\phi_1 + \nabla_\mu\phi_2\nabla_\nu\phi_2)\right] - \mathrm{U}(\psi,\phi_1,\phi_2)\right\}, \tag{2.10}$$

where $\psi \equiv 2\sqrt{2}\ln b$, $\phi_1 \equiv \tau_1$, and $\phi_2 \equiv \ln \tau_2$. Such action gives rise to the following set of equations:

$$6H_E^2 - \frac{1}{2}\left(\dot{\psi}^2 + e^{-2\phi_2}\dot{\phi_1}^2 + \dot{\phi_2}^2\right) = \frac{16\pi}{m_{pl}^2}U, \tag{2.11}$$

$$\ddot{\psi} + 3H_E\dot{\psi} = -\frac{16\pi}{m_{pl}^2}\frac{\partial U}{\partial \psi}, \tag{2.12}$$

$$\ddot{\phi_1} + 3H_E\dot{\phi_1} - 2\dot{\phi_1}\dot{\phi_2} = -\frac{16\pi}{m_{pl}^2}e^{2\phi_2}\frac{\partial U}{\partial \phi_1}, \tag{2.13}$$

$$\ddot{\phi_2} + 3H_E\dot{\phi_2} + e^{-2\phi_2}\dot{\phi_1}^2 = -\frac{16\pi}{m_{pl}^2}\frac{\partial U}{\partial \phi_2}, \tag{2.14}$$

and

$$4\dot{H}_E + \left(\dot{\psi}^2 + e^{-2\phi_2}\dot{\phi_1}^2 + \dot{\phi_2}^2\right) = 0. \tag{2.15}$$

Note that $H_E = (da_E/dt_E)/a_E$ is the Hubble constant in the Einstein's frame.

3. Casimir energy in $\mathbb{M}^{1+n} \times \mathbb{T}^p$

In this section, we will undergo the mathematical formulation to determine the Casimir energy, $\widehat{E}_{\text{cas}}$, associated with a scalar field of mass M in a $\mathbb{M}^{1+n} \times \mathbb{T}^p$ space. The fermionic degree of freedom will contribute to the Casimir energy with the same expression except for an extra minus sign. We then focus on the result from our phenomenological study ($n=3, p=2$).

3.1 Casimir-energy calculation

Let $V_n = L^n$ be the spatial volume of non-compact spacetime, and $V_p = l^p$ be the volume of compact space. If we assume $L \gg l$, the zero-point energy of scalar fields in $\mathbb{M}^{1+n} \times \mathbb{T}^p$ can be evaluated by

$$\widehat{E}_{\text{cas}} = \frac{1}{2}\left(\frac{L}{2\pi}\right)^n \sum_{n_i,n_j} \int_{-\infty}^{+\infty} d^n k \sqrt{\delta^{ab}k_a k_b + h^{ij}n_i n_j + M^2}, \tag{3.1}$$

where k_a; $a = 1,\ldots,n$ is the momentum in each non-compact spatial direction, $n_i \in \mathbb{Z}$; $i = 1,\ldots,p$ is the momentum number in each compact direction.

Using the property of integration in appendix A and changing variable of integration as $v = k^2/(h^{ij}n_in_j + M^2)$, we can express the Casimir energy as

$$\widehat{E}_{\rm cas} = \frac{1}{2}\left(\frac{L}{2\pi}\right)^n \frac{\pi^{n/2}}{\Gamma(n/2)} \sum_{n_i,n_j} (h^{ij}n_in_j + M^2)^{\frac{n+1}{2}} \int_0^\infty dv v^{\frac{n-2}{2}} \sqrt{1+v}. \qquad (3.2)$$

We can convert the integral into the Gamma function by using the formulae in appendix A; as a consequence, we obtain the Casimir energy in a simple form

$$\widehat{E}_{\rm cas} = \frac{1}{2}\left(\frac{2\pi}{L}\right)^{1+2s} \frac{\Gamma(s)}{\pi^{\frac{1+2s}{2}}\Gamma(-\frac{1}{2})} \sum_{n_i,n_j} (h^{ij}n_in_j + M^2)^{-s}; \quad s = -\frac{d-p+1}{2}. \qquad (3.3)$$

In our case, the compact space is $\mathbb{T}^2$ and h^{ij} is the inverse metric from eq. (2.2); therefore, our next task is to regularize the infinite summation in the eq. (3.3)

$$F\left(s; \frac{|\tau|^2}{b^2\tau_2}, -\frac{2\tau_1}{b^2\tau_2}, \frac{1}{b^2\tau_2}; M^2\right) = \sum_{n_1,n_2} \left(\frac{|\tau|^2}{b^2\tau_2}n_1^2 - \frac{2\tau_1}{b^2\tau_2}n_1n_2 + \frac{1}{b^2\tau_2}n_2^2 + M^2\right)^{-s}, \quad (3.4)$$

which is known as extended Chowla-Selberg zeta function [9]. It is worth noting that $V_p = l^2 = (2\pi b)^2$ in this case.

After a few steps of analytic manipulation by using Poisson resummation and property of the modified Bessel function, we obtain

$$\begin{aligned} F\left(s; \frac{|\tau|^2}{b^2\tau_2}, -\frac{2\tau_1}{b^2\tau_2}, \frac{1}{b^2\tau_2}; M^2\right) = b^{2s}\Bigg\{ & 2\tau_2^s \zeta_{\rm EH}(s; \tau_2 b^2 M^2) \qquad (3.5) \\ & + 2\sqrt{\pi}\frac{\Gamma(s-\frac{1}{2})}{\Gamma(s)} \tau_2^{1-s} \zeta_{\rm EH}\left(s - 1/2; \frac{b^2M^2}{\tau_2}\right) \\ & + \sum_{m,k=1}^{\infty} \frac{8\pi^s}{\Gamma(s)} \sqrt{\tau_2} k^{s-\frac{1}{2}} \frac{\cos(2\pi\tau_1 mk)}{\left(\sqrt{m^2 + \frac{b^2M^2}{\tau_2}}\right)^{s-\frac{1}{2}}} K_{s-\frac{1}{2}}\left(2\pi\tau_2 k\sqrt{m^2 + \frac{b^2M^2}{\tau_2}}\right)\Bigg\}, \end{aligned}$$

where the Epstein-Hurwitz zeta function $\zeta_{\rm EH}(s;q)$ is expressed as

$$\begin{aligned} \zeta_{\rm EH}(s;q) &= \frac{1}{2}\sum_{n\in Z}{}' (n^2+q)^{-s} \\ &= -\frac{q^{-s}}{2} + \frac{\sqrt{\pi}\Gamma(s-\frac{1}{2})}{2\Gamma(s)} q^{-s+\frac{1}{2}} + \sum_{n=1}^{\infty} \frac{2\pi^s q^{-s/2+1/4}}{\Gamma(s)} n^{s-\frac{1}{2}} K_{s-\frac{1}{2}}(2\pi n\sqrt{q}), \quad (3.6) \end{aligned}$$

where the prime at the first sum indicates that the term $n = 0$ is excluded. A similar expression which manifests the periodicity of the Casimir energy with respect to τ_1 is also given in ref. [12].

The expression serves as an analytic continuation of the Casimir energy where s is extended from positive to negative values. Inserting eq. (3.5) into eq. (3.3) and eliminating the infinite terms due to the pole of $\Gamma(s = -2)$ and $\Gamma(s - 1 = -3)$ in this case, we conveniently reached the regularized Casimir energy. The dropped divergent terms correspond

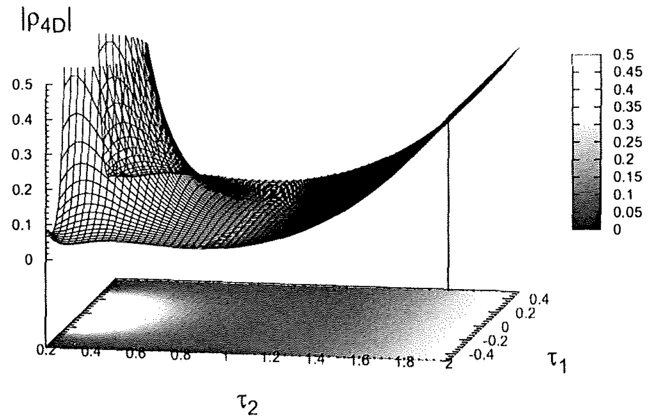


Figure 1: The magnitude of the Casimir energy density, $|\rho_{4D}|$, in four dimension per degree of freedom for $M = 5, b = 0.133$.

to the constant total energy and the constant energy density in the bulk. Both of them do not depend on any parameters of the torus and therefore can be safely eliminated from the physically relevant Casimir effects by renormalization. The final regulated Casimir energy density $\rho(h^{ij})$ in $(1 + 3)$-dimensional spacetime can then be expressed as

$$\begin{aligned}
\rho_{4D}(b^2, \tau_1, \tau_2) &= \frac{\widehat{E}_{\text{cas}}}{V_m} \\
&= -(4\pi^2 b^2)^s \Bigg\{ 2\tau_2^s (\tau_2 b^2 M^2)^{-\frac{s}{2}+\frac{1}{4}} \sum_{k=1}^{\infty} k^{s-\frac{1}{2}} K_{s-\frac{1}{2}}(2\pi k b M \sqrt{\tau_2}) \\
&+ 2\tau_2^{1-s} \left(\frac{b^2 M^2}{\tau_2}\right)^{-\frac{s}{2}+\frac{1}{2}} \sum_{k=1}^{\infty} k^{s-1} K_{s-1}\left(\frac{2\pi k b M}{\sqrt{\tau_2}}\right) \\
&+ 4\sqrt{\tau_2} \sum_{k,m=1}^{\infty} k^{s-\frac{1}{2}} \frac{\cos(2\pi\tau_1 k m)}{\left(\sqrt{m^2 + \frac{b^2 M^2}{\tau_2}}\right)^{s-\frac{1}{2}}} K_{s-\frac{1}{2}}\left(2\pi k \tau_2 \sqrt{m^2 + \frac{b^2 M^2}{\tau_2}}\right) \Bigg\}. \qquad (3.7)
\end{aligned}$$

In the case of massless scalar fields ($M = 0$), the Casimir energy density becomes

$$\begin{aligned}
\rho_{4D}(b^2, \tau_1, \tau_2) &= -(4\pi^2 b^2)^s \Bigg\{ \tau_2^s \pi^{s-\frac{1}{2}} \Gamma\left(\frac{1}{2} - s\right) \zeta(1-2s) + \tau_2^{1-s} \pi^{s-1} \Gamma(1-s) \zeta(2-2s) \\
&+ 4\sqrt{\tau_2} \sum_{m,k=1}^{\infty} \left(\frac{k}{m}\right)^{s-\frac{1}{2}} \cos(2\pi m k \tau_1) K_{s-\frac{1}{2}}(2\pi m k \tau_2) \Bigg\}. \qquad (3.8)
\end{aligned}$$

The Casimir density in (1+3+2) dimensions is given by $\rho_{6D} = \rho_{4D}/(2\pi b)^2$.

As it is pointed out in the work of Ponton and Poppitz [4]. Since the symmetry $\tau \to -1/\tau, \tau \to \tau + 1$ of the torus is preserved in the Casimir energy expression, it is sufficient to consider only the fundamental region where $\tau \geq 1, -1/2 \leq \tau_1 \leq 1/2$ of the shape moduli space. In the fundamental region, there are two minima and one saddle point of the magnitude $|\rho|$ of the Casimir energy density. The saddle point locates at $\tau_1 = 0, \tau_2 = 1$ and the two minima locate at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$. This is shown in figure 1.

3.2 Analysis for small bM

In the limit of $bM \ll 1$, we recalculate the Casimir energy by performing the binomial expansion with respect to small bM before regularization, and keep only the leading-order terms. It can be demonstrated that the process of regularizing each term after performing binomial expansion is NOT equivalent to the process of regularizing the whole expression at once if $s = -2$ is set beforehand. When we set $s = (1-d)/2 = -2$, the binomial expansion of eq. (3.4) gives only three terms with orders of $(bM)^0, (bM)^2$, and $(bM)^4$, whereas the regularization of the full expression before setting $s = -2$ as in eq. (3.5) ,which gives eq. (3.7) as a result, generically leads to an infinite series of bM, even after setting $s = -2$ in the final expression.

Without setting $s = -2$ before regularization, the precise dependence of the coefficients of the bM-binomial expansion to the moduli parameters τ_1, τ_2 will be determined. The small bM expansion is obtained subsequently.

We begin by replacing h^{ij} with the form of the inverse metric of $\mathbf{T}^2$ in eq. (3.3) and using Mellin transform (see appendix A)

$$\begin{aligned}
\widehat{E}_{\text{cas}} &= \frac{1}{2}\left(\frac{2\pi}{L}\right)^{1+2s} \frac{\Gamma(s)}{\pi^{\frac{1+2s}{2}}\Gamma(-\frac{1}{2})} \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty dt\ t^{s-1} e^{-\{\frac{1}{\tau_2 b^2}(|\tau|^2 n_1^2 - 2\tau_1 n_1 n_2 + n_2^2) + M^2\}t} \\
&= \frac{1}{2}\left(\frac{2\pi}{L}\right)^{1+2s} \frac{\Gamma(s)}{\pi^{\frac{1+2s}{2}}\Gamma(-\frac{1}{2})} \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty dv\ v^{s-1} e^{-\{\frac{1}{\tau_2}(|\tau|^2 n_1^2 - 2\tau_1 n_1 n_2 + n_2^2) + (bM)^2\}v} \\
&= \frac{1}{2}\left(\frac{2\pi}{L}\right)^{1+2s} \frac{b^{2s}}{\pi^{\frac{1+2s}{2}}\Gamma(-\frac{1}{2})} \sum_{j=0}^\infty \frac{(-1)^m}{m!} (bM)^{2j}\Gamma(s+j) \sum_{n_1,n_2\in\mathbb{Z}} \left(\frac{|\tau|^2}{\tau_2} n_1^2 - 2\frac{\tau_1}{\tau_2} n_1 n_2 + \frac{1}{\tau_2} n_2^2\right)^{-(s+j)},
\end{aligned} \tag{3.9}$$

where the second line is obtained by changing the dummy variable $v = t/b^2$, and the final line is obtained by expanding the Taylor series for $e^{-(bM)^2}$. We can determine the double summation in eq. (3.9) by using the result in eq. (3.4), (3.5); as a consequence, the Casimir energy density in five spatial dimensions takes the form,

$$\begin{aligned}
\rho_{6D}(b^2, \tau_1, \tau_2) = &-(4\pi^2 b^2)^{s-1} \sum_{j=0}^\infty \frac{(-1)^j}{j!} (bM)^{2j} \\
&\times \Big\{ 4\pi^j \sqrt{\tau_2} \sum_{m,k=1}^\infty \left(\frac{k}{m}\right)^{s+j-\frac{1}{2}} \cos(2\pi m k \tau_1) K_{s+j-\frac{1}{2}}(2\pi m k \tau_2) \\
&+ \pi^{s+2j-\frac{1}{2}} \tau_2^{s+j} \Gamma\left(\frac{1}{2} - s - j\right) \zeta(1-2s-2j) + \pi^{s+2j-1} \tau_2^{1-s-j} \Gamma(1-s-j) \zeta(2-2s-2j) \Big\}.
\end{aligned}$$

In the limit $bM \ll 1$ for $s = -2$, the Casimir energy density then becomes

$$\rho_{6D}(b^2, \tau_1, \tau_2) \simeq -\frac{1}{(4\pi^2 b^2)^3} \left\{ C_1 - C_2 (bM)^2 + C_3 (bM)^4 \right\} \tag{3.10}$$

where

$$C_1 \equiv \pi^{-\frac{5}{2}}\tau_2^{-2}\Gamma\left(\frac{5}{2}\right)\zeta(5)+\pi^{-3}\tau_2^3\Gamma(3)\zeta(6)+4\sqrt{\tau_2}\sum_{m,k=1}^{\infty}\left(\frac{m}{k}\right)^{\frac{5}{2}}\cos(2\pi mk\tau_1)K_{-5/2}(2\pi mk\tau_2),$$

$$C_2 \equiv \pi^{-\frac{1}{2}}\tau_2^{-1}\Gamma\left(\frac{3}{2}\right)\zeta(3)+\pi^{-1}\tau_2^2\Gamma(2)\zeta(4)+4\pi\sqrt{\tau_2}\sum_{m,k=1}^{\infty}\left(\frac{m}{k}\right)^{\frac{3}{2}}\cos(2\pi mk\tau_1)K_{-3/2}(2\pi mk\tau_2),$$

$$C_3 \equiv \frac{\pi}{2}\tau_2\Gamma(1)\zeta(2) + 2\pi^2\sqrt{\tau_2}\sum_{m,k=1}^{\infty}\left(\frac{m}{k}\right)^{\frac{1}{2}}\cos(2\pi mk\tau_1)K_{-1/2}(2\pi mk\tau_2). \tag{3.11}$$

In the next section, the total Casimir density for small bM and the full expression will be numerically compared. The true minimum of the potential, induced from the Casimir energy density located at a point $(\tau_1, \tau_2) = (\pm 1/2, \sqrt{3}/2)$, appears only when the full expression is evaluated.

4. Particle spectrum and effective potential for moduli fields

It is demonstrated in ref. [4] and ref. [3] that a careful mixing of massless and massive, bosonic and fermionic degrees of freedom of the bulk fields can lead to a Casimir energy density with local minimum with respect to the scale factor, b, of the compact extra dimensions. In the torus case with the shape moduli τ_1, τ_2, it can be shown that the true minimum of the mixed Casimir energy density (and thus the potential) locates at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$, in contrast to the case of undistorted torus considered in the previous work where the shape moduli are set to $\tau_1 = 0, \tau_2 = 1$.

The simplest model of the bulk fields in our $\mathbb{M}^{1+3} \times \mathbb{T}^2$ space consists of a massless boson, a massless fermion, a massive fermion with mass M, and a massive boson with mass λM. It was found that for the range $0.40 < \lambda < 0.42$ and $M = 5$, the mixed Casimir density has local minimum with respect to the scale factor b, and the moduli τ_1, τ_2. Since the mass of the boson is different from the mass of the fermion, this is the scenario where SUSY is broken in the bulk if it exists at higher scales. There is no particular reason for why the ratio of the masses of the massive boson and fermion took the specific value in this range. If it has anything to do with SUSY breaking, it is desirable that we are able to establish a SUSY breaking mechanism where this specific ratio of the masses λ could be explained or distinctively selected. From phenomenological point of view, it is desirable that these massless and small-mass bulk fields are sterile neutrinos for they can explain the smallness of neutrino masses in four dimensions. For further details, see ref. [13, 14].

An important issue in mixing bosonic and fermionic degrees of freedom to obtain the total Casimir energy density with a local minimum is the positivity of the energy density. Generally, the value of the total Casimir density at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$ is lower than the value at the saddle point $\tau_1 = 0, \tau_2 = 1$, for all range of λ. However, for certain ranges of λ (e.g. $\lambda \lesssim 0.407$), the density becomes negative around the true minimum and therefore violates the positive energy condition. A negative value of the density will not stabilize the dynamics and the size of the torus. We therefore choose the value $\lambda = 0.408$ for our

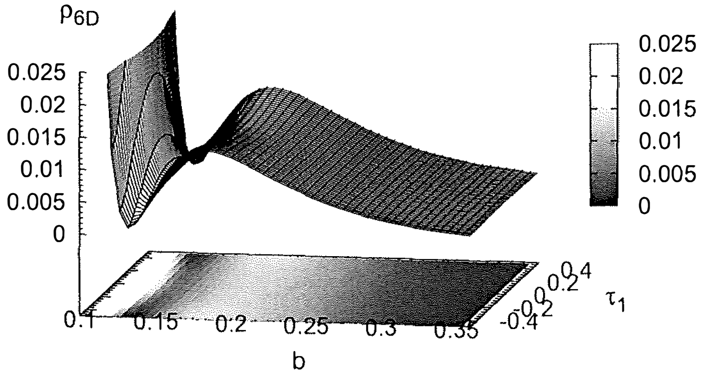


Figure 2: The total Casimir energy density in six dimension for mixture of massless and massive fields for $M = 5, \lambda = 0.408$, and $\tau = \sqrt{\tau_1^2 + \tau_2^2}$ is fixed to 1.

simulation of the cosmological dynamics. Figure 2 shows the total Casimir energy density for the spectrum of massless and massive particles mentioned above.

The plot of the total Casimir density in (1+3+2)-dimensional spacetime using the full expression, eq. (3.7), in comparison to the plot from the small bM approximation, eq. (3.10), is given in figure 3. The true minimum at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$ only exist in the full expression case. This can be understood considering $b_{\min} M \approx 0.67$ and is somewhat close to 1, resulting in a bad approximation of the expression due to higher powers of bM being neglected. It is therefore required that we use the full expression of the total Casimir energy density in the simulation of the cosmological dynamics.

5. Evidence of stability of the moduli space and cosmological dynamics

By numerically solving the field equations in section 2, the stabilization of the torus and the accelerated expansion of large 4-dimensional spacetime can be demonstrated to occur at the true minimum of the Casimir energy density in the moduli space. The point $\tau_1 = 0, \tau_2 = 1$ is a saddle point and it is an unstable equilibrium of the dynamics.

The rolling of the universe to the true minimum of the Casimir density is illustrated in figure 4–7. When the cosmological dynamics is initiated even within a small vicinity of the saddle point, $\tau_1 = 0, \tau_2 = 1$, of the Casimir energy density, it will roll down to the true minimum at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$ even with minimal amount of perturbations. This is shown in figure 4, 5. Observe that it tends to roll along the trail $\tau = 1$ in the moduli space.

When the tossing initial conditions are at a distant away from the saddle point and the true minimum, certain sets of the initial conditions still result in the stabilization of the torus moduli, τ_1, τ_2, and the scale factor, b, of the extra dimension as is shown in figure 6, 7. Naturally, as long as the Casimir energy density at the stabilized value is

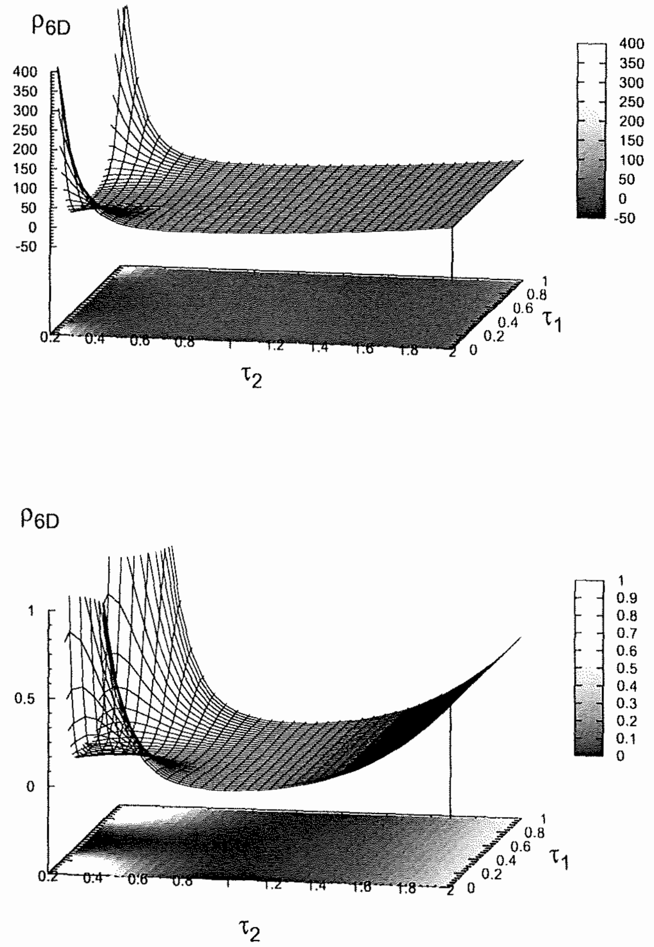


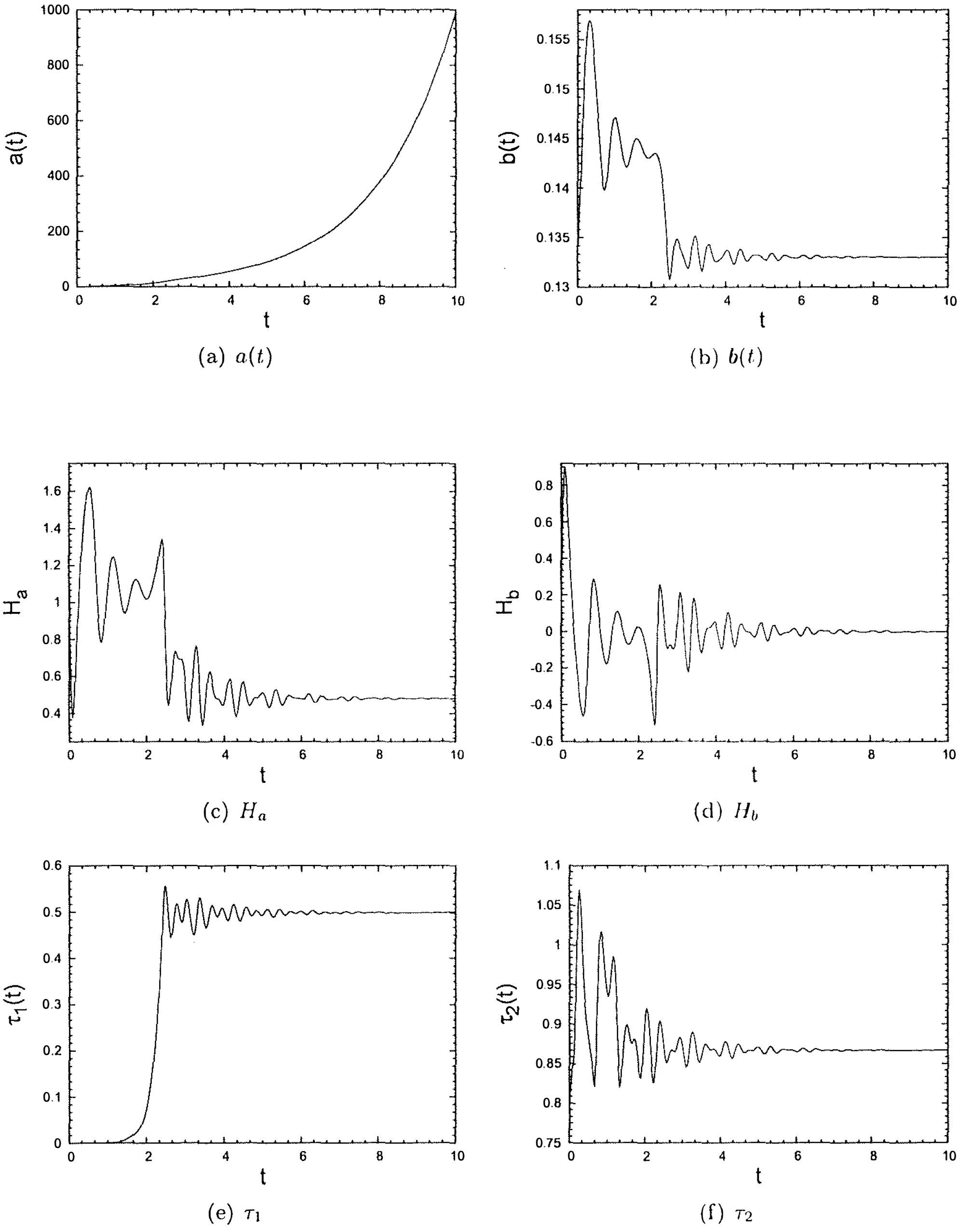
Figure 3: The Casimir energy density in six dimension from small bM approximation in the upper figure in comparison to the full expression in the lower figure. Both are evaluated at their corresponding b_{min}.

positive, the acceleration of the scale factor, a, of the 4-dimensional spacetime is guaranteed. The positive Casimir density serves as the positive cosmological constant.

A natural consequence of the Casimir energy that is independent of the scale factor, $a(t)$, of the large dimension is the fact that it leads to $w_a = -1$ for the pressure $p_a = w_a \rho$. For the pressure in the compact extra dimensions, we can start by considering $p_b = -\partial(\rho V_b)/\partial V_b = w_b \rho$, w_b of our Casimir energy density is then given by

$$w_b = -1 - \frac{b}{2\rho} \frac{\partial \rho}{\partial b} \tag{5.1}$$

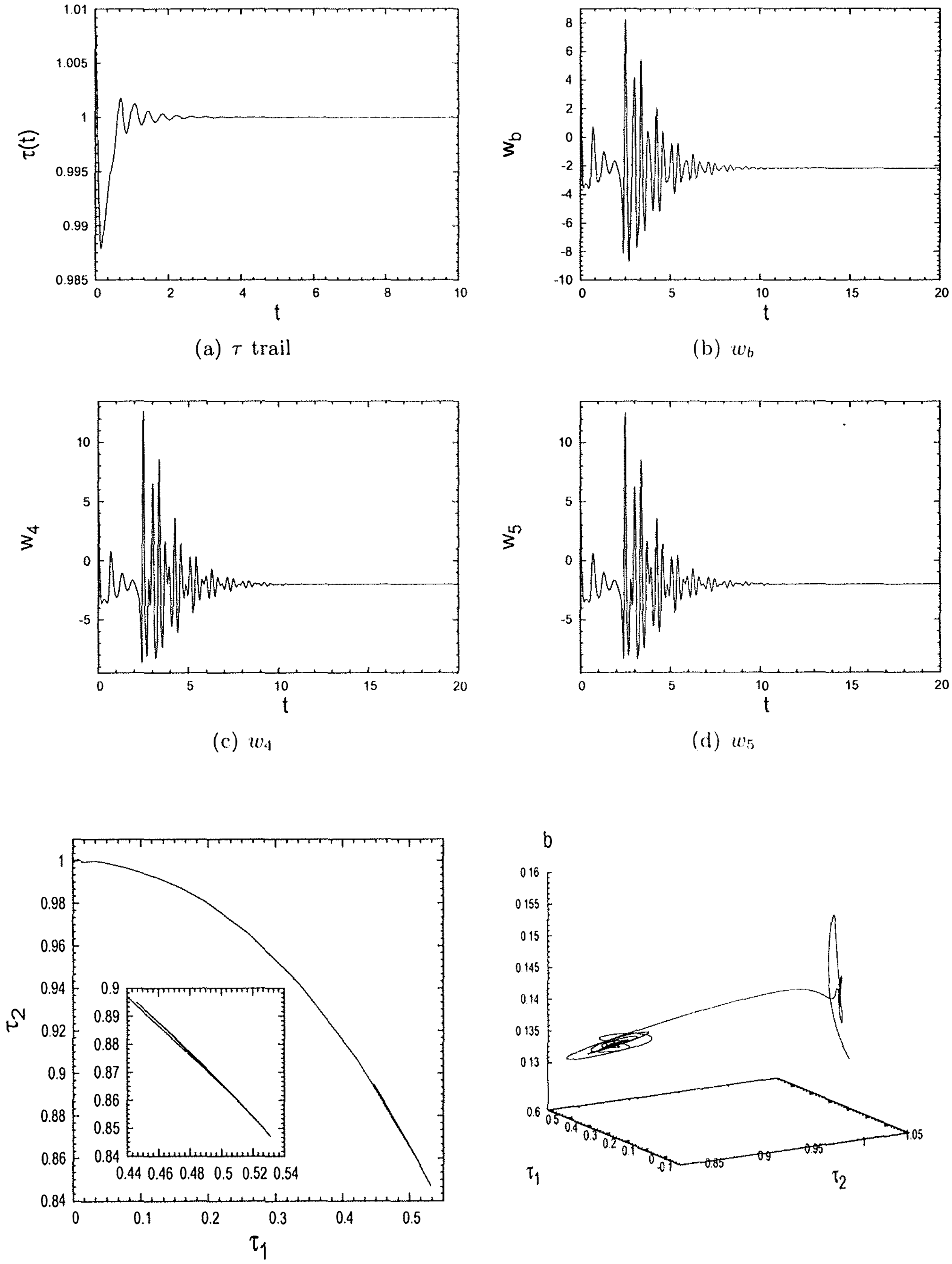
where ρ is the total Casimir energy density. Due to the dynamics of shape moduli (or



(a) $a(t)$ (b) $b(t)$ (c) H_a (d) H_b (e) τ_1 (f) τ_2

Figure 4: Cosmological dynamics when the universe is initially tossed very close to the saddle point $\tau_1 = 0, \tau_2 = 1$, it rolls along the trail $\tau = 1$ to the true minimum at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$.

Casimir "viscosity" in the compact space, see appendix B), the value of w_b at the stabilized radius at the true minimum is fractionally smaller than -2 (around -2.16) as is shown in figure 5.



(a) τ trail

(b) w_b

(c) w_4

(d) w_5

(e) rolling in shape moduli plane

(f) rolling in the landscape

Figure 5: Rolling dynamics from saddle point to the true minimum.

A more appropriate definition of physical pressures in the distorted torus is

$$p_K^* \equiv T_K^K, \tag{5.2}$$

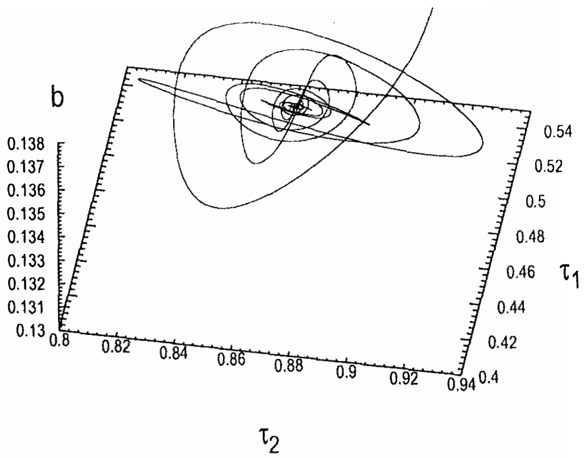


Figure 6: Rolling dynamics from other initial condition I.

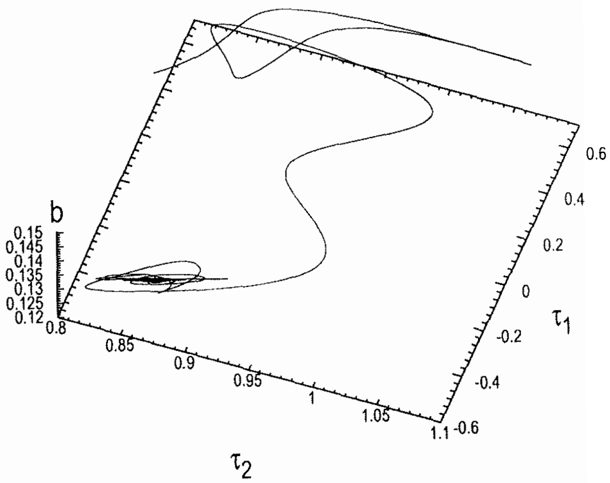


Figure 7: Rolling dynamics from other initial condition II.

where $K = 4, 5$. This definition gives the following expressions for $w_K = p_K^*/\rho$,

$$w_4 = -1 + \frac{b}{2\rho}\partial_b\rho\left(\frac{\tau_1^2 - \tau_2^2}{\tau_2^2}\right) + \frac{2\tau_1}{\rho}\partial_{\tau_1}\rho + \frac{1}{\rho}\left(\frac{\tau_2^2 - \tau_1^2}{\tau_2}\right)\partial_{\tau_2}\rho \tag{5.3}$$

$$w_5 = -1 + \frac{b}{2\rho}\partial_b\rho\left(\frac{\tau_1^2 - \tau_2^2}{\tau_2^2}\right) - \frac{1}{\rho}\left(\frac{\tau^2}{\tau_2}\right)\partial_{\tau_2}\rho. \tag{5.4}$$

By directly solving the equations of motion in six dimensions at the stabilized point where $\dot{H}_a = \dot{H}_b = H_b = \dot{\tau}_1 = \dot{\tau}_2 = 0$, it can be shown that $w_{4,5} = -2$, as is confirmed numerically in figure 5. It is interesting to note that the value of $w_{4,5}$ becomes -2 at both the saddle point and the true minimum where the dynamics is stabilized.

The difference of the two definitions of pressure originates from the *shear viscosity* induced by the Casimir energy in the off-diagonal components of the stress tensor. From the equations of motion of the 6-D universe with viscosities, eq. (B.10) in appendix B, shear viscosity at the stabilized point $\eta_b^{\rm stab}$ can be identified to be

$$\eta_b^{\rm stab} = \frac{3H_{a,stab}}{16\pi G} \tag{5.5}$$

$$= \frac{\rho_{6D,min}}{2H_{a,stab}} \tag{5.6}$$

where $H_{a,stab}$ is the Hubble constant of the expanding four dimensions at the stabilized point of the compactified space. Note that we can evaluate eq. (2.3), (B.8) and (B.9) at the stabilized point and use the definition of $\eta_b^{\rm stab}$ to analytically confirm the numerical results in which $w_{4,5} = -2$ at the stabilized point.

We should mention here that the time scale, t_s, of the simulated figures is given by

$$t_s = \frac{\sqrt{23}}{2\pi}\frac{m_{pl}}{b_{\rm min}}\, b_s^3, \tag{5.7}$$

where b_s is the scale of b, and $b_{\rm min} \simeq 0.1328 b_s$ as a result of numerical simulation. If we require that the stabilization time $\simeq 10t_s$ is less than the age of the universe, 10^{10} years, this will put constraint on the size $b_{\rm min}$ of the extra dimensions $\mathbb{T}^2$,

$$b_{\rm min} \lesssim 0.7\mu{\rm m}. \tag{5.8}$$

This is about few hundred times stronger than the constraints from table-top experiments [15].

It is interesting that in this kind of cosmological model, the constancy of the 4-dimensional gravitational constant, $G_4 = G/4\pi^2 b^2 = 1/m_{pl}^2$, up to the early times of the universe will give a very strong constraint on the size of the compactified extra dimensions. Any future observations of the universe from very early epoch could possibly put constraints on the inconstancy of the gravitational constant. Such constraints will put very strong limits on the size of compact extra dimensions in this kind of model where oscillatory behaviour is significant in the early times.

Another important aspect of this model is the relationship between the effective cosmological constant in 4-dimensional spacetime, $\Lambda_4 = 8\pi G_4 \rho_{4D,min}$, and the size of extra dimension, $b_{\rm min}$,

$$\Lambda_4 = 8\pi G_4 \rho_{4D,min} \tag{5.9}$$

$$= 3H_{E,stab}^2. \tag{5.10}$$

This leads to the typical value of $b_{\rm min} \approx 2.4\ \mu$m for $\rho_{\rm vac} \approx 10^{-11}\,{\rm eV}^4$. The value of the effective size of extra dimensions, $2\pi b_{\rm min} \approx 15\mu$m, yields the quantum gravity scale in the bulk, $M_* \approx 12\,{\rm TeV}$.

6. Conclusions and discussion

The stabilization of compact extra dimensions and the acceleration of the other 4-dimensional part of the spacetime can be simultaneously described by the dynamics of the Einstein field equations in the bulk spacetime. The acceleration of the 4-dimensional "universe" occurs naturally once the scale of the compact dimensions is stabilized and the density of the Casimir energy in the bulk becomes a (positive) constant at that stabilized value. As a result, the apparent positive "cosmological constant" that we seem to observe in the four dimensional visible universe is effectively induced. This is demonstrated beautifully in the work by Greene and Levin [3] when the Casimir density of the undistorted torus satisfies $w_a = -1, w_b = -2$ condition.

Shape moduli of the torus can be added to the model. The true minimum of the Casimir energy density of the torus with shape moduli is demonstrated to be located at $\tau_1 = \pm 1/2, \tau_2 = \sqrt{3}/2$. The cosmological dynamic shows that a minimally small perturbation to the saddle point rolls the universe down to the true minimum. Other initial conditions also suggest that the universe tends to roll around $\tau = 1$ contour to reach the true minimum. Note that it is also possible to stabilize the moduli at the saddle point $\tau_1 = 0$, $\tau_2 = 1$ but the initial conditions of the shape moduli fields must be fine-tuned so that $\tau_1 = 0, \dot{\tau_1} = 0$. Some extra-mechanisms such as Brandenberger-Vafa mechanism in string gas cosmology [16] is needed for this purpose. However, as it was pointed out in [17], the stabilized point $\tau_1 = \pm 1/2$ and $\tau_2 = \sqrt{3}/2$ is also the fixed point of T-duality and the the enhance symmetry point hence Brandenberger-Vafa mechanism could also set the initial value of the moduli precisely to be at the stabilized point.

The shear viscosity in the extra dimension is determined to be proportional to the Hubble constant at the stabilized point, $\eta_b = 3H_{a,stab}/16\pi G$. Through the Einstein field equations, this Hubble constant of the 4-D universe is determined by the value of the Casimir energy density at the stabilized point. The effective four dimensional cosmological constant is also given by $8\pi G\rho_{6D,min}$.

In this kind of model, there is a relationship between the size of the compact dimensions and the observed four dimensional cosmological constant. This remarkable connection is induced by the nature of Casimir energy density which depends on the size of the compact dimension, resulting in $\Lambda_4 \sim b_{\rm min}^{-6}$.

It is equally important to note that the constancy of the 4-D gravitational constant up to very early time of the universe will provide strong constraint on the size of extra dimension in this particular cosmological model which expresses oscillatory behaviour at the early times.

Acknowledgments

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A. Useful formulae

Phase space integration.

$$\int d^d k f(k) = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int k^{d-1} f(k) dk. \tag{A.1}$$

Poisson resummation.

$$\sum_{n\in\mathbb{Z}} f(n) = \tilde{f}(k) = \sqrt{2\pi} \sum_{m\in\mathbb{Z}} \tilde{f}(2\pi m), \tag{A.2}$$

where

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx. \tag{A.3}$$

If $f(x) = e^{-a(x+c)^2}$, then $\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\pi}{a}} e^{-\frac{k^2}{4a}+ikc}$.

Integral representation of Gamma function.

$$\Gamma(x) = \int_0^{+\infty} e^{-t} t^{x-1} dt. \tag{A.4}$$

The integral representation of the modified Bessel function of the second kind.

$$K_\nu(z) = \frac{1}{2}\left(\frac{z}{2}\right)^\nu \int_0^{+\infty} t^{-\nu-1} e^{-t-\frac{z^2}{4t}}\, dt, \tag{A.5}$$

where $|arg(z)| < \frac{\pi}{2}$, $\mathrm{Re}(z^2) > 0$.

Mellin transform.

$$z^{-s} = \frac{1}{\Gamma(s)} \int_0^\infty dt\, e^{-zt} t^{s-1}; \quad \mathrm{Re}(z) > 0, \quad \mathrm{Re}(s) > 0. \tag{A.6}$$

B. Energy momentum tensor of viscous fluid

Let $U^A = (1, 0, 0, 0, 0, 0)$ be the 6-velocity of the cosmic fluid in comoving coordinates. In terms of the projection tensor $h_{AB} = g_{AB} + U_A U_B$, the general energy momentum tensor of fluid with bulk viscosity ζ and shear viscosity η is given by:

$$T_{AB} = \rho U_A U_B + (p - \zeta\theta) h_{AB} - 2\eta\sigma_{AB}. \tag{B.1}$$

Here $\theta \equiv \nabla_A U^A$ is the scalar expansion and $\sigma_{AB} = h_A^C h_B^D \nabla_{(C} U_{D)} - \frac{1}{5} h_{AB}\theta$ is the shear tensor. By using metric defined in eq. (2.1) and (2.2), we can show that

$$T_0^0 = -\rho, \tag{B.2}$$
$$T_1^1 = T_2^2 = T_3^3 = p_a \tag{B.3}$$

$$T^4_4 = (p_b - \zeta_b\theta) - 2\eta_b\left[\frac{3}{5}(H_b - H_a) - \frac{\dot{\tau}_2}{2\tau_2} - \frac{\tau_1\dot{\tau}_1}{2\tau_2^2}\right] \tag{B.4}$$

$$T^5_5 = (p_b - \zeta_b\theta) - 2\eta_b\left[\frac{3}{5}(H_b - H_a) + \frac{\dot{\tau}_2}{2\tau_2} + \frac{\tau_1\dot{\tau}_1}{2\tau_2^2}\right] \tag{B.5}$$

$$T^4_5 = 2\eta_b\left[\frac{\tau_1\dot{\tau}_2}{\tau_2} + (\tau_1^2 - \tau_2^2)\frac{\dot{\tau}_1}{2\tau_2^2}\right] \tag{B.6}$$

$$T^5_4 = -\eta_b\frac{\dot{\tau}_1}{\tau_2^2} \tag{B.7}$$

Here we assume there is no viscosity in noncompact large dimensions ($\zeta_a = \eta_a = 0$). Einstein's equations, eq. (2.4)–(2.7), can be written in terms of bulk and shear viscosity as

$$\dot{H}_a + 3H_a^2 + 2H_aH_b = \frac{8\pi G}{4}\left\{\rho_{6D} + p_a - 2(p_b - \zeta_b\theta) + \frac{12}{5}\eta_b(H_b - H_a)\right\}, \tag{B.8}$$

$$\dot{H}_b + 2H_b^2 + 3H_aH_b = \frac{8\pi G}{4}\left\{\rho_{6D} - 3p_a + 2(p_b - \zeta_b\theta) - \frac{12}{5}\eta_b(H_b - H_a)\right\}, \tag{B.9}$$

$$\ddot{\tau}_1 + \left(3H_a + 2H_b - 2\frac{\dot{\tau}_2}{\tau_2}\right)\dot{\tau}_1 = 16\pi G\left\{\eta_b\dot{\tau}_1\right\}, \tag{B.10}$$

$$\frac{\ddot{\tau}_2}{\tau_2} + \frac{\dot{\tau}_1^2 - \dot{\tau}_2^2}{\tau_2^2} + 3H_a\frac{\dot{b}}{\tau_2} + 2H_b\frac{\dot{\tau}_2}{\tau_2} = 48\pi G\left\{\eta_b\frac{\dot{\tau}_2}{\tau_2}\right\}. \tag{B.11}$$

The conservation of energy is

$$\dot{\rho}_{6D} + 3H_a(\rho_{6D} + p_a) + 2H_b(\rho_{6D} + p_b) + \left(\frac{12}{5}\eta_b - 6\zeta_b\right)H_aH_b - \left(\frac{12}{5}\eta_b + 4\zeta_b\right)H_b^2 - \eta_b\left(\frac{\dot{\tau}_1^2}{\tau_2^2} + \frac{\dot{\tau}_2^2}{\tau_2^2}\right) = 0. \tag{B.12}$$

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Æther field, Casimir energy and stabilization of the extra dimension

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Abstract: In our five-dimensional cosmological model, we investigate the role of a Lorentz violating vector "æther" field on the moduli stabilization mechanism. We consider the case of a space-like æther field on a compact circle with Maxwell-type kinetic term. The Casimir energy of certain combinations of massless and massive bulk fields generates a stabilizing potential for the radius of the compact direction while driving the accelerated expansion in the non-compact directions. It is shown that the æther field can reduce the influence of the Casimir force and slow down the oscillation of the radion field. This property proves crucial to the stability of the extra dimension in the universe where non-relativistic matter is present. We speculate that this scenario might reveal a hidden connection between the dimensionality of spacetime and the spontaneous breaking of Lorentz symmetry.

Keywords: Cosmology of Theories beyond the SM, Classical Theories of Gravity, Space-Time Symmetries

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Contents

1 Introduction

Although there have been a lot of progresses on constructing phenomenologically viable models based on theories with extra spatial dimensions, some fundamental questions have not been completely solved. One of them is the moduli stabilization problem. The size and shape of compact space described by dynamical moduli fields have to be fixed in order to avoid any conflict with astronomical observations. In addition to these problems, we also face the challenges from cosmology in explaining the accelerated expansion of the universe. One possible solution for these problems may involve arguments based on anthropic principle. However, the search for an alternative solution is still going on, for example in [1].

Recently, it was suggested that Casimir energy from various field fluctuations in compact extra dimensions could play a crucial role in addressing these significant problems [2, 3]. Greene and Levin [3] argued that if the total Casimir energy is properly chosen, then it is possible, at least in the case of vacuum dominated universe, to stabilize the size of the extra dimensions and drive the accelerated expansion of the three non-compacted directions in which the Casimir energy plays the role of dark energy. The authors in [4] employed the calculation of Casimir energy in the non-trivial space, $\mathbb{M}^{1+3} \times \mathbb{T}^2$ and demonstrated that the shape of extra dimensions can also be stabilized by the same mechanism. Interestingly, predictions in this scenario such as radius of extra dimensions and quantum gravity scale in the bulk are in agreement with those from the large extra dimensions or ADD scenario [5, 6]. Hence, the moduli stabilization problem, dark energy problem and the hierarchy problem may possibly be explained in the single unified framework. However, as it was pointed out in [3], there are some crucial obstructions to realizing a phenomenological viable version of this scenario. One of them is that the extra dimension fails to stabilize if we include contribution from matter contents. During the matter dominant epoch, energy density of non-relativistic matter was the dominant contribution in the effective potential of the moduli fields and washed away the minimum of the effective potential. Although the minimum reappears in the vacuum dominated epoch, the moduli (i.e. radion field) has already passed the dynamical stable fixed point. This causes the extra dimension to expand and contradicts with our observation. Thus, it would be interesting to investigate whether this technical problem could be solved.

In this paper, we propose the new stabilization mechanism based on the Casimir energy and the existence of the Lorentz violating "æther" field in the compact direction. Starting with the simplest model with one extra dimension where the space-like æther field lives in the compact circle similar to the model considered in [7, 8], we claim that non-vanishing vacuum expectation value (vev) of the æther field would affect the dynamical equation of background moduli field. It can reduce the gradient of the radion's potential and slows down the oscillation frequency. This ensures stability of extra dimension although there is non-relativistic (dark) matter in the universe. As in the previous works, the Casimir energy of massless and massive fields embedded in five-dimensional space play a role of dark energy and drive the expansion of non-compact space as expected. Note that the effect of a time-like æther field on slowing down the expansion rate of the universe was pointed out in [9]. The effects of the æther fields on cosmological observable was studied in [10]. The authors in [11, 12] also studied the role of æther field on the stability of the extra dimension in the context of braneword scenario but in a different aspect.

It is important to state that we are aware of the stability issue for space-like æther field [13, 14] which may cause difficulty in the construction of a more realistic model of this scenario. However the interplay between the æther field and the dynamical moduli field in our model may shed some light on the connection between the dimensionality of spacetime and the violation of Lorentz symmetry. Perhaps nature allows us to observe only the large three-dimensional space that preserves Lorentz symmetry but conceals the Lorentz violating directions in the compact space.

This paper is organized as follows. In section 2, we start by reviewing the æther model in 5-dimensional spacetime. In section 3, we derive cosmological equations of motion in 5-dimensional spacetime with æther field and write down effective 4-dimensional equations of motion in the radion picture. Then we review the calculation of Casimir energy and extend to the case involving interaction between æther field and bulk fields in section 4. In section 5, we investigate the role of æther field in the context of stabilization of the extra dimension both in the vacuum dominated universe and in the universe with non-relativistic matter. Finally we summarize our results in section 6.

2 Æther field and its interactions

We start by considering a 5-dimensional flat spacetime with coordinates $x^a = (x^\mu, y)$ where $\mu = 0, \ldots, 3$ and with mostly plus metric signature. We assume that the fifth direction is compactified on a circle. Now we consider a toy model in which Lorentz symmetry is spontaneously broken by the æther field u^a i.e. a vector field with a non-vanishing expectation value. Most of the æther models contain kinetic term that makes their Hamiltonian unbounded from below and their stability is a subtle issue [13]. Here we consider the action with Maxwell-type kinetic term [8]

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{4}V_{ab}V^{ab} - \bar{\lambda}(u_a u^a - v^2) + \sum_i \mathcal{L}_i\right). \tag{2.1}$$

Here $V_{ab} = \nabla_a u_b - \nabla_b u_a$ has a familiar form to the field strength tensor of electromagnetism. However, the æther field u^a is not related to the electromagnetic vector field A^a and its dynamics does not respect U(1) gauge symmetry. In contrast, the second term in the above action enforces the æther field to have a constant norm

$$u^a u_a = v^2, \tag{2.2}$$

where $\bar{\lambda}$ acts as a Lagrange multiplier and we take $v^2 > 0$. In our unit u^a has dimension of mass$^{3/2}$. The sum $\mathcal{L}_i$ in (2.1) represent various interaction terms which couple the æther field to matter fields that we will discuss later in this section. If we neglect the interaction terms for the moment, the equations of motion for the æther field u^a can be written as

$$\nabla_a V^{ab} + v^{-2} u^b u_c \nabla_d V^{cd} = 0. \tag{2.3}$$

Any solutions for which $V_{ab} = 0$ will solve the equation of motion (2.3). In order to preserve Lorentz invariance in the 4-dimensional non-compact space, we choose the background solution such that the æther is a space-like vector field which has non-vanishing components along the extra fifth dimension,

$$u^a = (0, 0, 0, 0, v). \tag{2.4}$$

It is important to note that there is a subtle stability issue here. Although our aim is to investigate the role of the æther field on the stability of the extra dimension, the model of space-like æther field with Maxwell-type kinetic term itself is *unstable* [13]. However, for our purpose, we can consider it as *a toy model* and assume that there is some mechanism which stabilizes the æther field.

The energy-momentum tensor of the æther field $T_{ab}|_u$ takes the following form

$$T_{ab}|_u = V_{ac}V_b^c - \frac{1}{4}V_{cd}V^{cd}g_{ab} + v^{-2}u_a u_b u_c \nabla_d V^{cd}. \tag{2.5}$$

Note that properties of the æther field depend crucially on spacetime geometry. The flat space background solution in equation (2.4) gives $T_{ab}|_u = 0$. However, in curved spacetime, the æther field can give rise to non-vanishing energy momentum tensor for example a time-like æther field can produce energy density [9] while a space-like æther gives the stress components [14]. The case of an æther field oriented along the compact extra dimension was investigated in [8]. It was shown that such æther configuration can also give rise to non-vanishing energy momentum tensor. However, $T_{ab}|_u$ vanishes when the extra dimension is stabilized. We will review this result in the next section.

We now consider the effect of the interaction term $\sum_i \mathcal{L}_i$ in (2.1) which in general can include the terms corresponding to the æther field coupled to scalars, fermions and gravity. However, we will consider stabilization mechanism of the extra dimension involving Casimir energy of gravitons, bulk scalars and bulk fermions. We will ignore the bulk vector terms. Let us begin with the effect of the interaction of the æther with a real massive scalar field ϕ. The Lagrangian for the scalar field with the minimal coupling term is

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_a\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2\mu_\phi^2}u^a u^b \partial_a\phi\partial_b\phi, \tag{2.6}$$

where μ_ϕ is the coupling parameters with dimension of mass$^{3/2}$. The corresponding equation of motion for the scalar field takes the form [8]

$$\partial_a\partial^a\phi - m^2\phi = -\mu_\phi^{-2}\partial_a(u^a u^b\partial_b\phi). \tag{2.7}$$

Expanding the scalar field in Fourier modes $\phi \propto e^{ik_a x^a}$, we obtain the modified dispersion relation,

$$-k^\mu k_\mu = m^2 + (1+\alpha_\phi^2)k_5^2, \tag{2.8}$$

where the dimensionless parameter $\alpha_\phi = v/\mu_\phi$ is the ratio of the aether vev to the coupling μ_ϕ. Next we consider the fermion terms. The Lagrangian for fermionic field with the minimal coupling term can be written as [8]

$$\mathcal{L}_\psi = i\bar\psi\gamma^a\partial_a\psi - m\bar\psi\psi - \frac{i}{\mu_\psi^2}u^a u^b\bar\psi\gamma_a\partial_b\psi, \tag{2.9}$$

where μ_ϕ is the fermionic coupling constant with the unit of mass$^{3/2}$. In the same spirit as in the scalar field case, the corresponding modification of the dispersion relation for the fermionic case can be written as

$$-k^\mu k_\mu = m^2 + (1+\alpha_\psi^2)^2k_5^2, \tag{2.10}$$

where the dimensionless parameter $\alpha_\psi = v/\mu_\psi$. The form of this equation is different from the analogous equation in the bosonic case: i.e. the second term on the right-handed side increases by α_ψ^4 instead of α_ψ^2. Finally, we consider the æther field which couples non-minimally to gravity. This can be described by the action [8]

$$S_{GC} = \int d^5x\sqrt{-g}\left(\frac{M_*^3}{16\pi}R + \alpha_g u^a u^b R_{ab}\right), \tag{2.11}$$

where α_g is the dimensionless graviton coupling constant and M_* is the Planck mass in 5 dimensional space-time. By varying this action with respect to the metric tensor, we obtain the equation of motion $G_{ab} = 8\pi G T_{ab}|_{(GC)}$ with

$$T_{ab}|_{(GC)} = \alpha_g \Big(R_{cd}u^c u^d g_{ab} + \nabla_c \nabla_a (u_b u^c) + \nabla_b \nabla_c (u_a u^c) - \nabla_c \nabla_d (u^c u^d) g_{ab} - \nabla_c \nabla^c (u_a u_b) \Big), \tag{2.12}$$

where G is the 5-dimensional gravitational constant. Let us consider small fluctuation of the metric

$$g_{ab} = \eta_{ab} + h_{ab}. \tag{2.13}$$

Following the explanation in [8], the metric perturbation can be decomposed into

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \bar{\Phi}\eta_{\mu\nu}, \quad h_{55} = \bar{\Psi}, \tag{2.14}$$

where $\eta^{\mu\nu}\bar{h}_{\mu\nu} = 0$, $\bar{h}_{\mu\nu}$ presents the propagating modes of the gravitational wave, $\bar{\Phi}$ denotes the Newtonian gravitational field and $\bar{\Psi}$ is a component associated with the radion field describing the modes of the extra dimension. By setting $\bar{\Phi} = 0 = \bar{\Psi}$, and considering transverse waves, $\partial^\lambda \bar{h}_{\lambda\mu} = 0$, the gravitational equation of motion becomes

$$-\frac{1}{2}\partial^c \partial_c \bar{h}_{\mu\nu} = 8\pi \frac{\alpha_g v^2}{M_*^3} \partial_5^2 \bar{h}_{\mu\nu}. \tag{2.15}$$

Let us define $\tilde{\alpha}_g^2 = 16\pi \frac{\alpha_g v^2}{M_*^3}$. The above equation gives the modified dispersion relation for graviton

$$-k^\mu k_\mu = \left(1 + \tilde{\alpha}_g^2\right) k_5^2. \tag{2.16}$$

3 Cosmological dynamics and Æther field

3.1 Five-dimensional cosmological dynamics

In this section we consider cosmological dynamics of 5-dimensional spacetime by applying Einstein general relativity to the product space, between 4-dimensional FRW-type spacetime and a circle S^1. We assume the cosmological ansatz

$$ds^2 = -dt^2 + a(t)^2 dx^i dx^j \delta_{ij} + b(t)^2 dy^2, \tag{3.1}$$

where $i, j = 1, 2, 3$, $a(t)$ is the scale factor of non-compact 3-dimensional space, and $b(t)$ denotes the radius of the compact fifth direction. The coordinates on S^1 are $0 \leq y \leq 2\pi$. For our metric (3.1), the background solution for the equation of motion (2.3) can be written as

$$u^a = \left(0, 0, 0, 0, \frac{v}{b(t)}\right). \tag{3.2}$$

Using this background solution, the energy momentum tensor associated to the æther field defined in (2.5) can be written as

$$T^0{}_0|_u = -\frac{v^2}{2}H_b^2, \quad T^i{}_j|_u = \frac{v^2}{2}H_b^2\,\delta^i{}_j, \quad T^5{}_5|_u = -\frac{\ddot{b}}{b} + \frac{1}{2}H_b^2 - 3H_a H_b \tag{3.3}$$

We have defined the Hubble constants $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$, where dotted quantities represent the corresponding time derivative. As we mentioned in the previous section, $T_{ab}|_u = 0$ when the extra dimension is stabilized $\dot{b} = 0$. The fact that the æther field does not contribute to the energy density at the stabilized point implies that the æther field will not give any contribution to the effective potential of the radion. Hence other component such as Casimir energy is needed for stabilization of the extra dimension. However, as we shall see later on, the æther field can reduce the influence of the Casimir force. This property is important for stabilization mechanism when non-relativistic matter is present.

Let us assume that the total energy-momentum tensor $T_{ab}|_{\rm total}$ is decomposed into

$$T_{ab}|_{\rm total} = T_{ab}|_u + T_{ab}|_{GC} + T_{ab}|_\rho. \tag{3.4}$$

The contribution from non-minimally coupling to gravity $T_{ab}|_{GC}$ is defined in equation (2.12). The component $T^a_b|_\rho = diag(-\rho, p_a, p_a, p_a, p_b)$ represents contribution from Casimir energy [3]. Casimir energy density ρ plays the role of 5-dimensional cosmological constant. $p_a = -\rho$ and $p_b = -\frac{\partial(\rho 2\pi b)}{\partial(2\pi b)} = -\rho - b\partial_b\rho$ are the pressure density in non-compact and compact direction respectively. By substituting $T_{ab}|_{\rm total}$ into the Einstein field equation, we get the 5-dimensional cosmological equations of motion

$$3H_a^2 + 3H_aH_b = 8\pi G\left(\rho + \frac{1}{2}v^2H_b^2\right), \tag{3.5}$$

$$3\frac{\ddot{a}}{a} - 3H_aH_b = -8\pi G\Big\{\rho + p_b - (1-2\alpha_g)v^2A\Big\}, \tag{3.6}$$

$$3\frac{\ddot{b}}{b} + 9H_aH_b = 8\pi G\Big\{\rho + 2p_b - 3p_a - 2(1-2\alpha_g)v^2A\Big\}, \tag{3.7}$$

where $A = (\frac{\ddot{b}}{b} + 3H_aH_b)$.

3.2 Dynamics in the radion picture

Since we are interested in our observed universe, it is useful to analyze the cosmological dynamics by considering 4-dimensional effective field theory. The equations of motion (3.5)–(3.7) can be obtained by varying the 5-dimensional Einstein-Hilbert action

$$S_{5D} = \int d^5x\sqrt{-g}\left(\frac{M_*^3}{16\pi}R - \frac{1}{4}V_{ab}V^{ab} + \alpha_g u^a u^b R_{ab} - V(b)\right). \tag{3.8}$$

$V(b)$ denotes the potential term in 5-dimensional spacetime. Note that we omit the Lagrange multiplier term. For simplicity, we will set $\alpha_g = 0$ in this section and this will not affect our main results. Let us start with KK-dimensional reduction of the above action from 5 to 4-dimensional spacetime. Then, in order to make the resulting effective action in the canonical form, we apply Weyl rescaling $g_{\mu\nu E} = \Omega g_{\mu\nu}$ $(\mu, \nu = 1, \ldots, 3)$ and define the new time variable $dt_E = \sqrt{\Omega}dt$, $a_E(t_E) = \sqrt{\Omega}a(t)$; $\Omega = 2\pi b M_*^3/m_{pl}^2$. Note that m_{pl} is the Planck mass in 4-dimensional spacetime defined via the relation $m_{pl}^2 = (2\pi b_{\rm min})M_*^3$ where $b_{\rm min}$ denotes the stabilized radius of extra dimension. Thus $\Omega = 1$ at $b = b_{\rm min}$. The effective action takes the form

$$S_{4D} = \int d^4x\sqrt{-g_E}\left\{\frac{m_{pl}^2}{16\pi}R_E - \frac{1}{2}g_E^{\mu\nu}\nabla_\mu\Psi\nabla_\nu\Psi - \frac{1}{2}\frac{m_{pl}^2}{M_*^3 b_{\rm min}^2}e^{-2\frac{\sqrt{16\pi}}{\sqrt{3}m_{pl}}\Psi}V_\mu V^\mu - U(\Psi)\right\}, \tag{3.9}$$

where $U(\Psi) = 2\pi b\Omega^{-2}V(b)$ is the 4-dimensional effective potential. Here we define the radion field $\Psi = \frac{m_{pl}}{\sqrt{16\pi}}\sqrt{3}ln(b/b_{min})$ and $V_\mu = V_{\mu 5} = \nabla_\mu u_5$. By using the background solution in (3.2), the above 4-dimensional action can be rewritten as

$$S_{4D} = \int d^4x\sqrt{-g_E}\left\{\frac{m_{pl}^2}{16\pi}R_E - \frac{1}{2}(1+\alpha^2)g_E^{\mu\nu}\nabla_\mu\Psi\nabla_\nu\Psi - U(\Psi)\right\}, \tag{3.10}$$

where we define the dimensionless parameter $\alpha^2 = \frac{16\pi v^2}{3M_*^3}$. This action gives rise to the following set of equations:

$$H_E^2 = \frac{8\pi}{3m_{pl}^2}\left\{U(\Psi) + \frac{1}{2}(1+\alpha^2)\left(\frac{d\Psi}{dt_E}\right)^2\right\}, \tag{3.11}$$

$$\frac{d^2\Psi}{dt_E^2} + 3H_E\frac{d\Psi}{dt_E} = -\frac{1}{(1+\alpha^2)}\frac{\partial U}{\partial\Psi}. \tag{3.12}$$

Note that $H_E = (da_E/dt_E)/a_E$ is the Hubble constant in the Einstein frame. As we will explain later, the factor $1/(1+\alpha^2)$ in the right-handed side of equation (3.12) weakens the effect of the potential gradient $-\partial U/\partial\Psi$ and it is crucial for stabilization mechanism of the radion field Ψ. To make contact with previous section, we note that the energy-momentum tensor associated with 5-dimensional action in (3.8) gives the relations

$$\rho = \frac{\Omega}{Gm_{pl}^2}U\ , \quad 2\rho + p_b = -\frac{\Omega}{Gm_{pl}^2}\left(b\partial_b U\right). \tag{3.13}$$

4 Æther field and Casimir energy

We will start this section by reviewing the mathematical formulation to determine the Casimir energy for a scalar field, $\widehat{E}_{\rm cas}$, and then investigating the effect of æther coupling to the Casimir energy. First, we consider Casimir energy of a non-interacting scalar field of mass, m, in $\mathbb{M}^{1+n}\times S^1$ spacetime by following [15, 16]. We keep the number of non-compact spatial directions to be n for the moment and will set $n=3$ at the end of our calculation. This scalar field obeys the free Klein-Gordon equation,

$$(\partial_a\partial^a - m^2)\phi = 0. \tag{4.1}$$

The scalar field satisfies the periodic boundary condition in the compact direction, $\phi(y=0) = \phi(y=2\pi)$. Its associated dispersion relation can be written as

$$-k^\mu k_\mu = m^2 + \frac{\tilde{n}^2}{b^2}, \tag{4.2}$$

where, $\tilde{n}\in\mathbb{Z}$ is the momentum number in the compact direction. Then, the total vacuum energy contributing to Casimir energy can be written as

$$\widehat{E}_{\rm cas} = \frac{1}{2}\left(\frac{L}{2\pi}\right)^n\int d^nk\sum_{\tilde{n}}\sqrt{k^2+m^2+\frac{\tilde{n}^2}{b^2}}, \tag{4.3}$$

where $V_n = L^n$ is the spatial volume of non-compact spacetime. Using the fact that $\int f(k)d^nk = 2\pi^{n/2}/\Gamma(n/2)\int k^{n-1}f(k)dk$, we obtain

$$\widehat{E}_{\text{cas}} = \frac{1}{2}\left(\frac{L}{2\pi}\right)^n \frac{2\pi^{n/2}}{\Gamma(n/2)}\int k^{n-1}dk\sum_{\tilde{n}}\sqrt{k^2+m^2+\frac{\tilde{n}^2}{b^2}}, \tag{4.4}$$

$$= \frac{1}{2}\left(\frac{2}{L}\right)^{2s+1}\frac{\Gamma(s)}{\Gamma(-1/2)}b^{2s}\pi^{(2s+1)/2}\sum_{\tilde{n}}\left((bm)^2+\tilde{n}^2\right)^{-s}, \tag{4.5}$$

where we define $s = -(n+1)/2$. Let us consider the massless case, $m = 0$. By using the zeta function regularization procedure, the Casimir energy density per one bosonic degree of freedom for massless scalar field can be written as

$$\widehat{\rho}_{\text{cas}}^{\text{massless}} = \frac{\widehat{E}_{\text{cas}}}{V_n 2\pi b} = \frac{\Gamma(-2s+1)}{\Gamma(-1/2)}2^{2s}b^{2s-1}\pi^{3s-1}\zeta(-2s+1), \tag{4.6}$$

where ζ denotes the zeta function and we take $2\pi b$ to be the volume of compact dimension. For the massive case, we apply the Chowla-Selberg zeta function [16] in our regularization procedure and obtain the Casimir energy density per one degree of freedom for the massive scalar field:

$$\widehat{\rho}_{\text{cas}}^{\text{massive}} = -2(2\pi b)^{2s-1}(mb)^{(1-2s)/2}\sum_{n=1}^{\infty}n^{(2s-1)/2}K_{(1-2s)/2}(2\pi bmn), \tag{4.7}$$

where $K_\nu(x)$ is the modified Bessel function. The fermionic degrees of freedom will contribute to the Casimir energy density with the same expression except for an extra minus sign.

Let us consider the case that a scalar field couples to an æther field with a coupling constant α_ϕ. In the previous section, we showed that interaction with the æther field transforms the usual dispersion relation (4.2) into its modified version (2.8). Accordingly, the Casimir energy will be written as

$$E_{\text{cas}}(\alpha_\phi) = \frac{1}{2}\left(\frac{L}{2\pi}\right)^n\int d^nk\sum_{\tilde{n}}\sqrt{k^2+m^2+\left(1+\alpha_\phi^2\right)\frac{\tilde{n}^2}{b^2}},$$

$$= \frac{1}{2}\left(\frac{L}{2\pi}\right)^n\frac{2\pi^{n/2}}{\Gamma(n/2)}\int k^{n-1}dk\sum_{\tilde{n}}\sqrt{k^2+m^2+\left(1+\alpha_\phi^2\right)\frac{\tilde{n}^2}{b^2}},$$

$$= \left(1+\alpha_\phi^2\right)^{(n+1)/2}\frac{1}{2}\left(\frac{L}{2\pi}\right)^n\frac{2\pi^{n/2}}{\Gamma(n/2)}\int k'^{n-1}dk'\sum_{\tilde{n}}\sqrt{k'^2+m'^2+\frac{\tilde{n}^2}{b^2}}, \tag{4.8}$$

where we rescale k and m in such a way that $k^2 = (1+\alpha_\phi^2)k'^2$ and $m^2 = (1+\alpha_\phi^2)m'^2$. By comparing $E_{\text{cas}}(\alpha_\phi)$ with the non-interacting Casimir energy $\widehat{E}_{\text{cas}}$, we see that the æther coupling rescales the Casimir energy and scalar mass by factors $(1+\alpha_\phi^2)^{(n+1)/2}$ and $(1+\alpha_\phi^2)^{-1/2}$ respectively. Thus, we can immediately write down the Casimir energy density per one bosonic degrees of freedom with the æther coupling α_ϕ as

$$\rho^{\text{massless}}_{\text{boson}}(\alpha_\phi) = \frac{\Gamma(-2s+1)}{\Gamma(-1/2)} \frac{2^{2s} b^{2s-1} \pi^{3s-1}}{(1+\alpha_\phi^2)^s} \zeta(-2s+1), \tag{4.9}$$

$$\rho^{\text{massive}}_{\text{boson}}(\alpha_\phi) = -\frac{2(2\pi b)^{2s-1}}{(1+\alpha_\phi^2)^s} \left(\frac{mb}{\sqrt{1+\alpha_\phi^2}}\right)^{\frac{(1-2s)}{2}} \sum_{\tilde{n}=1}^{\infty} \tilde{n}^{(2s-1)/2} K_{(1-2s)/2}\left(\frac{2\pi mb\tilde{n}}{\sqrt{1+\alpha_\phi^2}}\right), \tag{4.10}$$

for contributions from massless and massive scalar fields respectively. Other bosonic degrees of freedom contribute to the total Casimir energy in a similar way but with different coupling constants and masses. For example, the graviton in $(n+2)$-dimensional spacetime has $\frac{1}{2}(n+2)(n-1)$ bosonic degrees of freedom with the æther coupling α_g and mass $m=0$. Using the modified dispersion relation for graviton (2.16), we can show that the graviton contributes $\frac{(n+2)(n-1)}{2}\rho^{\text{massless}}_{\text{boson}}(\tilde{\alpha}_g)$ to the total Casimir energy density. Recall that, apart from modification of graviton's Casimir energy, non-minimal coupling of the æther field to gravity can also affect the dynamical evolution through the energy momentum tensor $T_{ab}|_{(GC)}$.

For fermion case, we use the modified dispersion relation for the fermionic field in (2.10). We can show that the associated Casmir energy density per one fermionic degree of freedom, for both massless and massive case, is in the similar form of those from bosinic degrees of freedom with the over all minus sign and $(1+\alpha_\phi^2) \to (1+\alpha_\psi^2)^2$. The Casimir energy densities for one degree of freedom of massless and massive fermion can be written respectively as

$$\rho^{\text{massless}}_{\text{fermion}}(\alpha_\psi) = -\frac{\Gamma(-2s+1)}{\Gamma(-1/2)} \frac{2^{2s} b^{2s-1} \pi^{3s-1}}{(1+\alpha_\psi^2)^{2s}} \zeta(-2s+1), \tag{4.11}$$

$$\rho^{\text{massive}}_{\text{fermion}}(\alpha_\psi) = \frac{2(2\pi b)^{2s-1}}{(1+\alpha_\psi^2)^{2s}} \left(\frac{mb}{1+\alpha_\psi^2}\right)^{(1-2s)/2} \sum_{n=1}^{\infty} n^{(2s-1)/2} K_{(1-2s)/2}\left(\frac{2\pi mbn}{1+\alpha_\psi^2}\right). \tag{4.12}$$

The total Casimir energy density can be written in terms of sum over all degrees of freedom:

$$\rho = N_b \rho^{\text{massless}}_{\text{boson}}(\tilde{\alpha}_g) + N_f \rho^{\text{massless}}_{\text{fermion}}(\alpha_\psi) + \tilde{N}_b \rho^{\text{massive}}_{\text{boson}}(\alpha_\phi) + \tilde{N}_f \rho^{\text{massive}}_{\text{fermion}}(\alpha_\psi), \tag{4.13}$$

where N_b (N_f) and $\tilde{N}_b$ ($\tilde{N}_f$) are the numbers of bosonic (fermionic) degrees of freedom for massless and massive fields respectively. The nature of the total Casimir energy density depends on the relative magnitude of N_b, N_f, $\tilde{N}_b$ and $\tilde{N}_f$.

In our model, $N_b \geqslant 5$ since, at least, the graviton is always present and it has five physical degrees of freedom in five-dimensional spacetime ($n=3$). The compact fifth direction would not be stable if there is only the graviton field in the bulk, i.e. it will collapse to Planck size, due to the negative Casimir energy associated with quantum fluctuations of the gravitation fields. Therefore, it is natural to add more positive contribution to the Casimir energy by assuming that there are fermions in the bulk. However, we cannot create the minimum of ρ by including only the massless fermionic fields i.e. the Casimir force is attractive for $N_f < N_b$ and becomes repulsive when $N_f > N_b$. Hence $N_f = N_b$

Particles	Degrees of freedom	Mass	Coupling constant
a bulk graviton field	$N_b = 5$	0	$\tilde{\alpha}_g = (16\pi\frac{\alpha_g v^2}{M_*^3})^{1/2}$
a massless bulk fermion field	$N_f = 8$	0	α_ψ
a massive bulk fermion field	$\tilde{N}_f = 8$	m_f	α_ψ
8 massive bulk scalar fields	$\tilde{N}_b = 8$	$m_s = \lambda m_f$	α_ϕ

Table 1. The particle spectrum in the bulk, their degrees of freedom, their mass and the coupling constants characterizing their interaction with the æther field. For simplicity, we assume the universal fermionic coupling α_ψ for both massless and massive Dirac fermion.

does not give us any stable fixed point. We should expect a minimum to be produced if we include massive fermionic degree of freedom [17]. This can be explained qualitatively as the following. Let us consider Casimir energy of a fermion with mass M. For the region where $b \ll 1/M$, the vacuum energy should have the same form as in the massless case. In particular, for $N_f > N_b$, the net Casimir force will be repulsive. In the other region where $b \gg 1/M$, the contribution from the massive fermion mode is negligible compared to that of the graviton and the total Casimir force becomes attractive. Hence, there must be a stable fixed point between these two regions.

In this paper, we consider a toy model where the particle spectrum in the bulk consists of a bulk graviton, a massless Dirac fermion, a massive Dirac fermion with mass m_f, and eight massive scalars with equal masses $m_s = \lambda m_f$. Here λ is the mass ratio. The presence of the massless fermion and the massive scalars is to ensure that the vacuum energy at the minimum has positive value, i.e. $\rho_{\min} > 0$. We summarize the particle content in the bulk in table 1.

There is no unique choice of bulk particle spectrum for this purpose, the other combinations of bulk fields, for example in ref. [3], can probably create minimum for the vacuum energy. Our particular choice is convenient for investigating the effect of the æther-matter interactions on the Casimir energy of the bulk fields. Note that we do not attempt to justify the existence of these bulk fields phenomenologically because we want to demonstrate that stability of the extra dimension could be achieved, if these particles are present in the bulk. The Dirac fermions in five dimensional spacetime have eight physical degrees of freedom. For more realistic models which have chiral fermion on the brane, we can impose the orbifold reflection symmetry: $y \to -y$ on the compact direction. However, this will not affect our main results on stabilization of the extra dimension. We will ignore this issue for simplicity.

5 Effects of The Æther field on stabilization of the extra dimension

5.1 Stabilization in vacuum dominated universe

We first consider the universe where there is no non-relativistic matter and the Casimir energy density is the dominant contribution. Let us start with the case where there is no

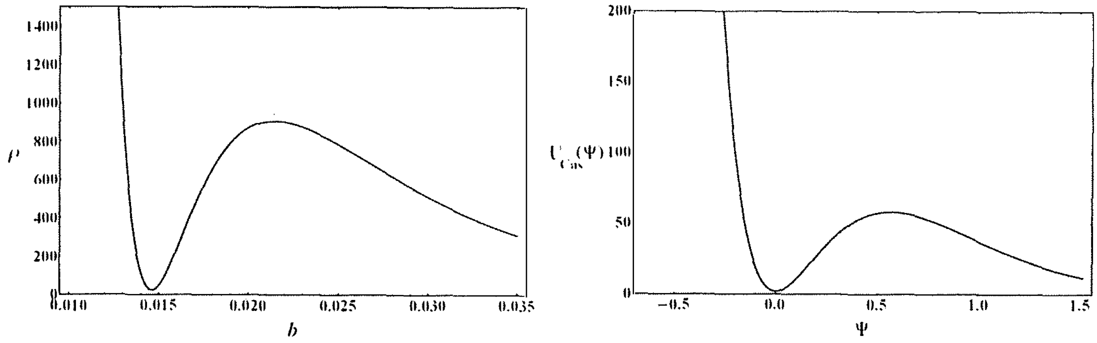

Figure 1. Left: Casimir enery density ρ is presented in the y-axis in units of $(m_f/40)^5$. The x-axis is the radius of the fifth dimension b in units of $40/m_f$. Right: The 4-dimensional effective Casimir potential $U_{\text{Cas}} = 2\pi(b^2_{\min}/b(\Psi))\rho(\Psi)$ in unit of $(m_f/40)^4$ is presented in the y-axis. The x-axis denotes Ψ in the unit of m_{pl}. Here we set $\alpha = \alpha_\phi = \alpha_\psi = \alpha_g = 0$.

interaction between æther field and bulk matter i.e. setting $\alpha_\phi = \alpha_\psi = \alpha_g = 0$. The plot of the total Casimir energy density in 5-dimensional spacetime ρ using the expression in equation (4.13) and particle spectrum in table 1, with the mass ratio $\lambda = 0.516$, and its corresponding 4-dimensional effective $U_{\text{Cas}} = 2\pi(b^2_{\min}/b(\Psi))\rho(\Psi)$ is given in figure 1. The local minimum of Casimir energy density, $\rho_{\min} = 23.4316(m_f/40)^5$ is located at $b_{\min} = 0.01461(40/m_f)$. In order to get the positive minimum and the radion can stabilize, we must choose the value of λ from a very narrow range, $0.516 \leq \lambda \leq 0.527$. By solving the 5-dimensional equations of motion (3.5)–(3.7) numerically, we can demonstrate that the extra dimension is stabilized at the radius $b_{\min}$ as shown in figure 2. Notice that we set the expansion time scale to be in the unit of Hubble time $t_H = H_{a0}^{-1} = \sqrt{3m^2_{pl}/8\pi\rho_c} \approx 10^{10}$ years. The critical density ρ_c can be written in terms of the minimum value of Casimir energy density $\rho_{\min}$ as $\rho_c = (1 + 0.24/0.76)(2\pi b_{\min})\rho_{\min}$.

By comparing 4-dimensional effective Casimir energy density $\rho^{(4)}_{\min} = (2\pi b_{\min})\rho_{\min}$ with the observed value of energy density for dark energy, $\rho^{(4)}_{\text{obs}} \approx (2.3 \times 10^{-3}\ eV)^4$, we get $m_f \approx 4.18 \times 10^{-2}\ eV$. Then, the radius of extra dimension $b_{\min} \sim 13.96\ eV^{-1} \sim 2.75 \times 10^{-6}\ m$. This leads to the quantum gravity scale in the bulk, $M_* \approx 1.19 \times 10^9\ GeV$. Note that we do not attempt to address the mass hierarchy problem in this paper. In order to compare with the ADD brane world scenario, it is better to be demonstrated with 6-dimensional models as shown in [3]. Since our aim is to study the role of æther fields on stabilization of the extra dimension, the 5-dimensional model is good enough for our purpose. We will leave the mass hierarchy problem for future works.

The role of æther field on dynamical evolution of the extra dimension is also illustrated in figure 2. We can see that, as the value of v increase, the moduli field b feel less potential gradient. Its oscillation frequency and amplitude decrease. In our caculation v is in the unit of $(m_f/40)^{5/2}t_H$. From the previous paragraph $m_f = 4.18 \times 10^{-2}\ eV$, this gives 1 unit of v is equivalent to $(1.02 \times 10^8\ GeV)^{3/2} \approx (0.09M_*)^{3/2}$. At very high v, for example $v = 100$ or approximately $\approx (9M_*)^{3/2}$, the period of oscillation is so long that b reaches equilibrium before showing any oscillating behavior. The scale factor b rolls slowly to its stable fixed point.

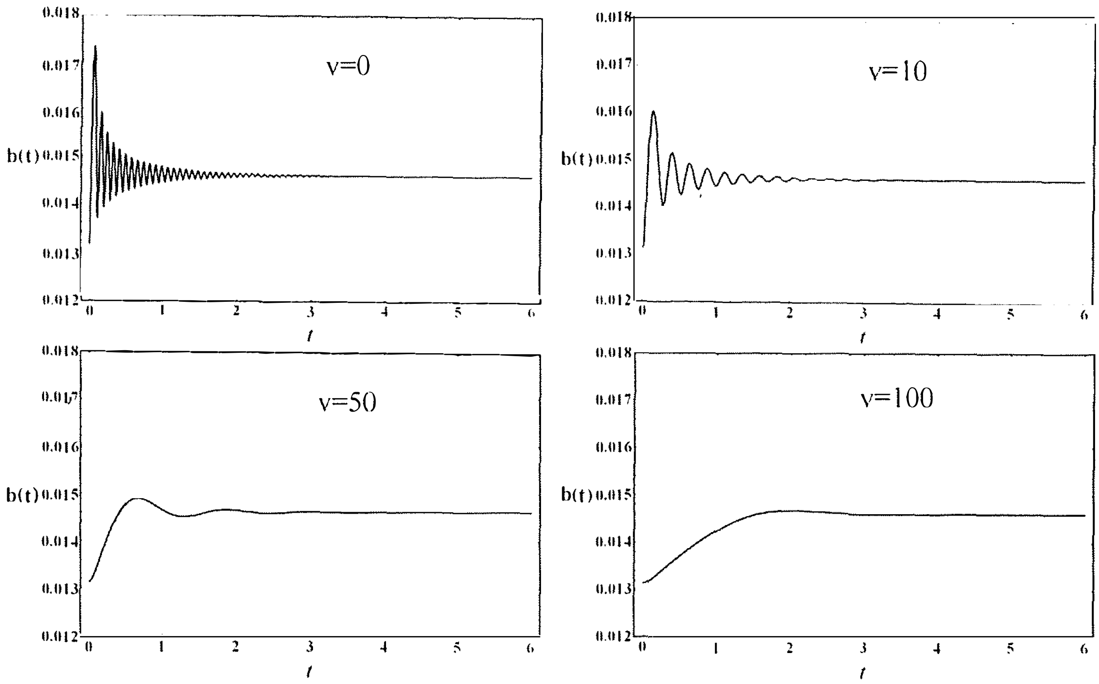

Figure 2. The dynamics of the scale factor $b(t)$ for the compact direction as a function of time with different values of parameter v. In the absence of æther field $v = 0$, $b(t)$ shows oscillation behavior around its critical value $b_{\rm min}$ before stabilizing at this value. Non-vanish value of v reduces the influence of Casimir force. As the value of v increases, the oscillation frequency and amplitude decrease. If the vev of the æther field is large enough, for example $v = 100$, oscillation behavior disappears. The extra dimension evolves smoothly to its stable fixed point. The time variable t is presented in the unit of Hubble time t_H. The time for stabilization to occur is around $\sim 6t_H$. The condition for stabilization of b is $\frac{\delta b}{b} \leqslant 10^{-5}$.

Let us consider the situation where æther field couples to the bulk matters. As we discussed earlier, interactions with the æther field reduce the effective mass of the bulk fields. For example, the effective mass for scalar field of mass m_s would be

$$m^2_{s(eff)} = m_s^2 \left(1 + \alpha_\phi^2\right)^{-1} . \tag{5.1}$$

This will alter the shape of the potential as we demonstrate in figure 3. Interestingly, there is an advantage of coupling the bulk fields with the æther field. It seems that we get the wider range of parameter space for the mass ratio λ that allows positive minima $\rho_{\rm min} > 0$, i.e $0.05 \lesssim \lambda \lesssim 0.80$.

5.2 Stabilization in the universe with non-relativistic matter

In this section we consider the role of æther field on stabilizing mechanism of the extra dimension in the more realistic model of our universe i.e. a model containing non-relativistic matter. Let us fist demonstrate the destabilizing effect due to non-relativistic matter by following Greene and Levin in Ref [3]. We assume that there is matter living in the bulk.

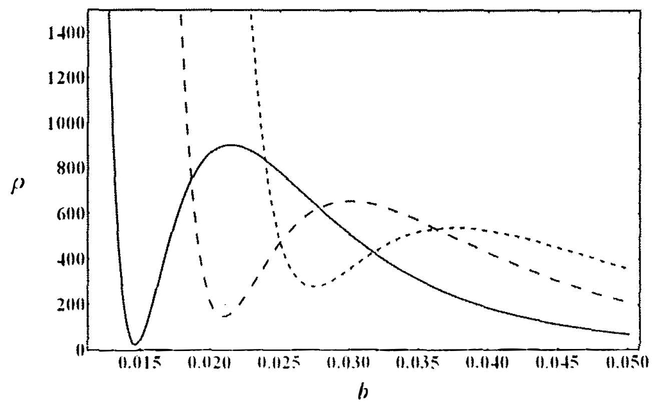

Figure 3. Interactions between bosons/fermions and æther field can affect the Casimir energy. In this figure, we fix the mass ratio $\lambda = 0.516$. The Casimir enery density ρ is presented in the y-axis in units of $(m_f/40)^5$. The solid, long dashed and short dashed line denote the Casimir energy density when the coupling constant $(\alpha_\phi, \alpha_\psi) = (0.0, 0.0), (1.0, 0.644)$, and $(1.5, 0.897)$ respectively. The value of $b_{\rm min}$ and $\rho_{\rm min}$ increase as we increase the value of the coupling constants. The shape of the potential well gets shallower as the coupling increases. We set $\tilde{\alpha}_g = \alpha_\phi$ for simplicity.

This can be done by adding a 5-dimensional matter term into the 5-dimensional action in equation (3.8)

$$S = S_{5D} + \int d^5x\sqrt{-g}\mathcal{L}_{\rm matter}. \tag{5.2}$$

This is equivalent to adding the matter energy density $\rho_m \propto 1/2\pi a^3 b$ into the 5-dimensional cosmological equations of motion (3.5)–(3.7). The matter energy density ρ_m includes contribution from baryonic matter and cold dark matter. By comparing with the observational data and supposing that all dark matter is cold, the matter density today ρ_{m0} is roughly 26% of the total energy density of the universe. The Casimir energy density will be responsible for the other 74% of the total energy density today in the form of dark energy, $(\rho_{\Lambda 0})$. Thus, we have the relation, $\rho_{m0} = (2.6/7.4)\rho_{\Lambda 0}$. Note that the energy density of dark energy in our 4-dimensional observed universe today can be written in terms of the minimum of 5-dimensional Casimir energy density and the stabilized radius of the extra dimension as $\rho_{\Lambda 0} = \rho_{\rm min}(2\pi b_{\rm min}) = (2.3\times 10^{-3}eV)^4$. By using $(a_0/a) = 1+z$, a_0 is the scale factor today and z is the red-shift, we get

$$\rho_m = \frac{2.6}{7.4}\rho_{\rm min}\left(\frac{b_{\rm min}}{b}\right)(1+z)^3. \tag{5.3}$$

In this case, the 5-dimensional equations of motion (3.5)–(3.7) become

$$3H_a^2 + 3H_aH_b = 8\pi G\left(\rho + \rho_m + \frac{1}{2}v^2H_b^2\right), \tag{5.4}$$

$$3\frac{\ddot{a}}{a} - 3H_aH_b = -8\pi G\Big\{\rho + \rho_m + p_b - (1-2\alpha_g)v^2A\Big\}, \tag{5.5}$$

$$3\frac{\ddot{b}}{b} + 9H_aH_b = 8\pi G\Big\{\rho + \rho_m + 2p_b - 3p_a - 2(1-2\alpha_g)v^2A\Big\}. \tag{5.6}$$

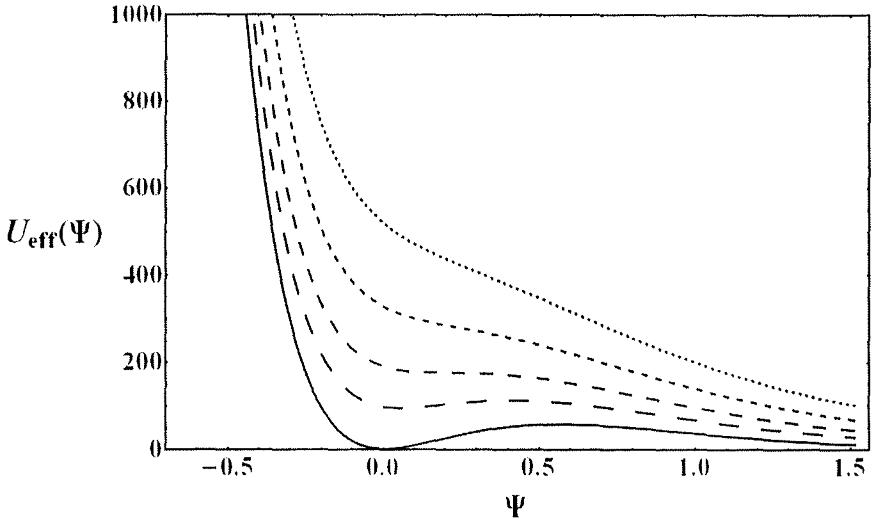

Figure 4. This is the plot of the effective potential $U_{\text{eff}}(\Psi)$ at red shift $z = 0.0, 7.0, 9.0, 11.0, 13.0$ in the unit of $(m_f/40)^4$ and $\lambda = 0.516$. The local minimum of $U_{\text{eff}}(\Psi)$ no longer exists when the red shift is increased.

From equation (5.6), the stability conditions ($\dot{H}_b = 0$, $H_b = 0$ and $A = 0$) require

$$p_b = -2\rho - \frac{1}{2}\rho_m. \tag{5.7}$$

By using equation (5.5) and requiring that $\frac{\ddot{a}}{a} > 0$ when b is stabilized, we get the constraint $\rho_m < 2\rho$. This is the same constraint that we have for a $(3+1)$-dimensional vacuum dominated universe. Thus, as pointed out in [3], this model describes the $(3+1)$-dimensional vacuum dominated universe when the extra dimension is stabilized.

By using the conservasion of the energy-momentum tensor in four and five dimensions, we can easily show that the radion field will be driven toward the minimum of the 4-dimensional effective potential

$$U_{\text{eff}} = U_{\text{Cas}} + \frac{m_{pl}^2 G}{\Omega}\frac{\rho_m}{4} = U_{\text{Cas}} + \frac{\rho_m^{(4)}}{4}\left(\frac{b_{\text{min}}}{b}\right)^2, \tag{5.8}$$

where we define the 4-dimensional matter density $\rho_m^{(4)} = \rho_m(2\pi b) = \frac{2.6}{7.4}\rho_{\text{min}}\,(2\pi b_{\text{min}})\,(1+z)^3$ which is a function of $(1+z)^3$ and does not depend on the radius of the extra dimension b.

Numerical results for the effective potential U_{eff} are illustrated in figure 4. Here we choose $\lambda = 0.516$, and ignore the interaction terms by setting $\alpha_\phi = \alpha_\psi = \alpha_g = 0$. At $z = 0$, the presence of non-relativistic matter would lift up the minimum of U_{eff} slightly. However, at early time, high red-shift, the $1/b^2$-term in (5.8) becomes dominant and destroys the presence of the minimum. This effect will drive b to expand even though there is a minimum today since the radion field Ψ has already rolled pass the minimum and cannot get back to the stable point. Notice that this effect is the same if matter is confined to the brane.

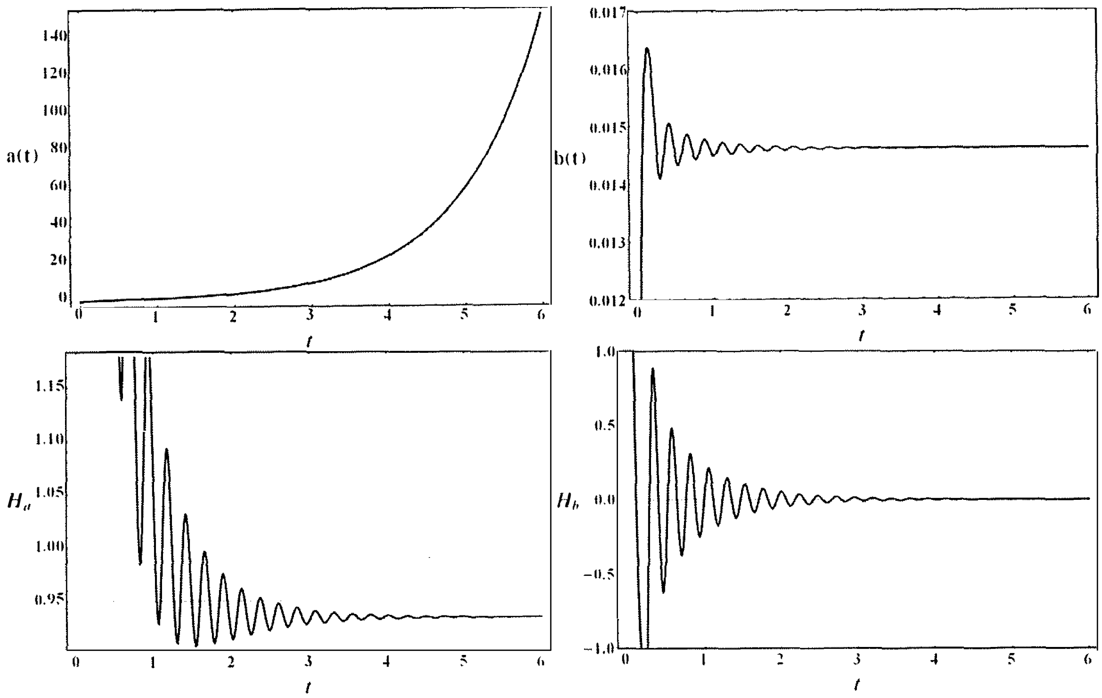

Figure 5. These graphs illustrate cosmological dynamics of the universe which includes non-relativistic matter content and æther field in the extra dimension. Left: The scale factor a (upper) and the Hubble constant for the non-compact dimensions H_a (lower) as the function of time. The fact that H_a oscillates indicating the period of deceleration and acceleration before it settles down to the constant value and a enters a de Sitter phase. Right: The scale factor b (upper) and the Hubble constant for the compact extra dimension H_b (lower) as the function of time. H_b oscillates between positive and negative region before settles down to zero. The extra dimension is stabilized although non-relativistic matter is present.

this effect is proved to be crucial for stabilization of the extra dimension. If the vev of æther field is of the order of the 5-dimensional Planck mass, $v \sim O(M_*^{3/2})$, it can slow down the evolution of the moduli field such that there is enough time to create the minimum for the effective potential.

In this paper we assume homogeneous and isotropic distribution of non-relativistic matter. However, local matter distributions might perturb the radion and knock it over the minimum, causing the (local) catastrophic expansion of the fifth dimension. In [3], it was also noted that the minimum of the potential well is generally not deep enough to prevent the quantum tunneling of the radion. At this stage, it is not clear whether these two difficulties can be solved by the new mechanism. These aspects of instability in the presence of the æther are still open questions.

Note that the constancy of the 4-dimensional cosmological constant up to very early epoch of the universe will post strong constraint on the size of the extra dimension. The oscillation behavior of the moduli field may contradict with astronomical observations. In order to construct a more realistic cosmological model of this scenario, the extra dimension should reach its stable fixed point before the present time i.e. $t_{\rm stab} \lesssim t_{\rm age}$ which require fine tuning of many parameters. The new possible solution is that we assume very high value of v so that the oscillation of b has a very long period. The moduli will evolve smoothly with no oscillating behavior. We can choose the value of v such that the size of the extra dimension changes so slowly and it cannot alter the results of the Big Bang model.

Another interesting idea is to imagine that the universe started with a very symmetric state which all spatial dimensions are compactified with the equal radius, for example, topologically an 4-dimensional torus. In general, the Casimir energies in this compact universe generate stabilized potential for the radius of all directions. On the other hand, at the very early time, the energy density of matter and radiation will be the dominant contribution. This will destabilize the moduli fields and all directions will become large. However, if the Lorentz symmetry is spontaneously broken in some direction i.e. there is a non-vanishing æther field pointing in the fifth direction, it will slow down the dynamics of moduli field associated to the broken direction. The broken direction will be compactified at stabilized radius while the unbroken directions are allowed to expand. This cosmological scenario may establish a connection between the dimensionality of spacetime and the violation of Lorentz symmetry. We leave this issue for future investigation.

Acknowledgments

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