

Chapter 2

BACKGROUND

2.1. Point Kinetics

The point kinetics equations can be derived from the general diffusion equation: [4]

$$\frac{1}{v} \frac{\partial \Phi}{\partial t} = [M - k_{\text{eff}}^{-1} F] \Phi, \quad (2.1.1)$$

where M neutron elimination operator (absorption, scattering and leakage),
 F production operator (fission and delayed neutrons),
 k_{eff} effective multiplication factor,
 Φ neutron flux, and
 v neutron scalar speed,

by introducing the following flux factorization:

$$\Phi(\mathbf{r}, E, t) = p(t)\Psi(\mathbf{r}, E), \quad (2.1.2)$$

where $p(t)$ time-dependent amplitude,
 \mathbf{r} spatial coordinate,
 E neutron energy,
 Ψ the fixed flux shape, usually corresponding to the initial steady-state conditions.

The amplitude $p(t)$ is arbitrarily normalized to unity at a chosen power level, which is, for convenience, the full power (FP) in this thesis:

$$p_{\text{FP}} = 1 \text{ or } 100\% \quad (2.1.3)$$

Thus, the point kinetics equations can be written as follows:

$$\frac{dp}{dt} = \frac{\rho(t) - \beta}{\Lambda} p(t) + \sum_{i=1}^N \lambda_i \zeta_i(t) + s(t) \quad (2.1.4)$$

$$\frac{d\zeta_i}{dt} = -\lambda_i \zeta_i(t) + \frac{\beta_i}{\Lambda} p(t) \quad (2.1.5)$$

where $p(t)$ is defined as in (2.1.2, 2.1.3), and can represent the neutron power in percentage of reactor full power, %FP,

$\zeta_i(t)$ weighted precursor concentration in delayed neutron group i ,

$i = 1, 2, \dots, N$

λ_i decay constant of precursor of delayed neutron group i ,

β effective delayed neutron fraction,

β_i fraction of delayed neutrons in group i , $\sum_{i=1}^N \beta_i = \beta$,

Λ mean life time of prompt neutrons ℓ divided by k_{eff} , $\Lambda = \frac{\ell}{k_{\text{eff}}}$,

$s(t)$ weighted external source,

$\rho(t)$ dynamic reactivity.

The initial steady-state conditions can be obtained by equating the right hand sides of (2.1.4) and (2.1.5) to zero. Thus, the initial weighted precursor concentration of each delayed neutron group is:

$$\zeta_i(0) = \rho_0 \frac{\beta_i}{\lambda_i}. \quad (2.1.6)$$

The initial source strength and reactivity, however, are either

for critical reactor: $s(0) = 0$ and $\rho(0) = 0$, or $(2.1.7)$

for subcritical reactor: $\rho(0) = \rho_0 < 0$ and $s(0) = s_0 = -\rho_0 \rho_0$. $(2.1.8)$

The dynamic reactivity in (2.1.4) is considered as a function of a number of global parameters:

$$\rho(t) = \rho_{\text{ex}} + \rho_{\text{ctrl}}(t) + \rho_{\text{Xe}}(t) + \rho_{\text{temp}}(t) + \rho_{\text{void}}(t), \quad (2.1.9)$$

where ρ_{ex} is the fuel loadexcess reactivity assumed to be unchanged for a short time of operation. The other time-dependent items are as follows:

Control Reactivity:

The control reactivity, ρ_{ctrl} , is the negative reactivity of all control elements inserted into the reactor core to compensate the excess reactivity of the fuel, and to control the reactor. For the rod form of control elements, the control reactivity at any given time is the total reactivity of all control rods. Integral reactivity, $\rho(z)$, versus position of each control rod is usually determined from the regular rod calibrations. Thus,

$$\rho_{\text{ctrl}}(t) = \sum_{i=1}^N \rho_i(z_i) \quad (2.1.10)$$

where z_i position of the i -th control rod,

$\rho_i(z)$ integral reactivity of the i -th control rod vs. its position, and

N total number of control rods.

Xenon Poison:

It is known that several fission products are strong neutron absorbers, among which Xe-135 is the most significant, especially, for relatively long time operations. The xenon feedback, $\rho_{Xe}(t)$, can be calculated from solving the buildup equations of I-135 and Xe-135:

$$\frac{dN_I}{dt} = \gamma_I p - \lambda_I N_I \quad (2.1.11)$$

$$\frac{dN_{Xe}}{dt} = (\gamma_{Xe} p + \lambda_I N_I) - (\lambda_{Xe} + \sigma_{Xe} \Phi) N_{Xe} \quad (2.1.12)$$

where N_I and N_{Xe} weighted numbers of nuclides I-135 and Xe-135,
 γ_I and γ_{Xe} fission yields,
 λ_I and λ_{Xe} decay constants,
 σ_{Xe} microscopic absorption cross-section of Xe-135,
 Φ average thermal neutron flux.

Temperature Effects:

Increase of temperature in the reactor core has three effects that cause changes in reactivity. There are the density effect due to thermal expansion, the Doppler broadening effect due to widening of the resonance in U-238, and the neutron spectrum effect due to increase of molecular speed. Each of them affects the neutron cycle more or less, and will immediately introduce a positive or negative reactivity - the fast reactivity feedback.

The temperature feedback, $\rho_{temp}(t)$, is a function of fuel and coolant/moderator temperatures with reference to an arbitrary temperature T_0 which can be, for convenience, the initial core temperature at critical zero power level:

$$\rho_{temp}(t) = \int_{T_0}^{T_f} \left(\frac{\partial \rho}{\partial T_f} \right)_{T_w} dT_f + \int_{T_0}^{T_w} \left(\frac{\partial \rho}{\partial T_w} \right)_{T_f} dT_w \quad (2.1.13)$$

where T_f average fuel temperature, and

T_w average coolant/moderator temperature in the core.

It is customary to define the fuel temperature coefficient, α_f , as:

$$\alpha_f = \left(\frac{\partial \rho}{\partial T_f} \right)_{T_w = \text{const}} \quad (2.1.14)$$

and the coolant/moderator temperature coefficient, α_w , as

$$\alpha_w = \left(\frac{\partial \rho}{\partial T_w} \right)_{T_f = \text{const}} \quad (2.1.15)$$

The void feedback, $\rho_{\text{void}}(t)$, is due to void formation occurring if any boiling takes place in the core. In this work, only subcooled liquid is considered and no boiling is allowed, therefore, the void feedback is ignored. However, for accident analysis, the void feedback must be taken into account because the negative reactivity introduced by void formation from subcooled nucleate boiling, although very small in volume, can have quite significant reactivity effects.



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2.2. Thermal-Hydraulics

2.2.1. Conservation Equations [5]

The general conservation equation for any field variable is written as

$$\frac{D}{Dt} \iiint_V \psi dV = \iiint_V \Gamma dV + \iint_S \Pi \cdot \bar{n} dS \quad (2.2.1)$$

where $\frac{D}{Dt}$ time substantial derivative = change due to time variations plus change due

to movement at the velocity of the field variable, ψ ,

\iiint volume integral over the arbitrary volume V ,

\iint surface integral over the surface S of the volume V ,

ψ a continuous field variable such as mass, momentum, energy, etc.,

Γ net sum of local sources and local sinks of the field variable ψ within the volume V ,

Π net sum of local sources and local sinks of the field variable ψ on the surface S ,

\bar{n} unit vector normal to the surface.

Using Reynold's Transport Theorem:

$$\frac{D}{Dt} \iiint_V \psi dV = \iiint_V \frac{\partial \psi}{\partial t} dV + \iint_S \psi \bar{\omega} \cdot \bar{n} dS \quad (2.2.2)$$

where $\frac{\partial}{\partial t}$ local time derivative, and

$\bar{\omega}$ velocity of the field variable,

we can transform (2.2.1) into:

$$\iiint_V \frac{\partial \psi}{\partial t} dV = - \iint_S \psi \bar{\omega} \cdot \bar{n} dS + \iiint_V \Gamma dV + \iint_S \Pi \cdot \bar{n} dS \quad (2.2.3)$$

In words, this states that the change in the conserved field variable ψ in the volume V is due to surface flux, plus sources and minus sinks.

Using Gauss' Divergence Theorem:

$$\iint_S \bar{A} \cdot \bar{n} dS = \iiint_V \nabla \cdot \bar{A} dV \quad (2.2.4)$$

where \bar{A} any vector, such as velocity or heat flux, and

∇ Del operator, (eg. $\nabla = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k}$),

we can rewrite (2.2.3) as:

$$\frac{\partial \psi}{\partial t} = -\nabla \cdot \psi \bar{\omega} + \Gamma + \nabla \cdot \Pi \quad (2.2.5)$$

This is the distributed or microscopic form. Equation (2.2.3) is the macroscopic or “lumped” form. They are equivalent and one can move freely back and forth between the two forms as long as the field variables are continuous.

Conservation of Mass:

Interpreting $\psi = \rho \equiv$ the fluid density, $\Gamma = 0$, and $\Pi = 0$ (there is no volume generation and no surface flux), we have

$$\text{distributed form:} \quad \frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \bar{\omega} \quad (2.2.6)$$

$$\text{lumped form:} \quad \frac{\partial M}{\partial t} = \sum_j W_j \quad (2.2.7)$$

where M mass in the volume V : $M = \iiint_V \rho dV$

W mass flow into or out of the volume V : $\sum_j W_j = -\iint_S \rho \bar{\omega} \cdot \bar{n} dS = \sum_j \rho_j \omega_j A_j$

Conservation of Momentum:

Substituting $\psi = \rho \bar{\omega} \equiv$ the momentum per unit volume,

$\Gamma = \rho \bar{f} \equiv$ the momentum generated by volumetric forces, and

$\Pi = [\sigma] \equiv$ the surface stress tensor,

into (2.2.5), we have:

$$\text{distributed form:} \quad \frac{\partial \rho \bar{\omega}}{\partial t} + \nabla \cdot \rho \bar{\omega} \bar{\omega} = \nabla \cdot [\sigma] + \rho \bar{f} \quad (2.2.8)$$

where \bar{f} is the long range or body force (i.e. gravity). The stress tensor, $[\sigma]$, can be split into the normal and shear components:

$$[\sigma] = -P[I] + [\tau]$$

where P is the pressure, $[I]$ is the unity tensor and $[\tau]$ is the shear tensor. Then, (2.2.8) becomes:

$$\frac{\partial \rho \bar{\omega}}{\partial t} + \nabla \cdot \rho \bar{\omega} \bar{\omega} = -\nabla \cdot P[I] + \nabla \cdot [\tau] + \rho \bar{f} \quad (2.2.9)$$

$\nabla \cdot [\tau]$ is usually evaluated by an empirical correlation.

Assuming one dimensional flow, such as in a pipe with the length L and flow inlet and outlet areas A_{in} and A_{out} , the lumped form can be written as:

$$V \frac{\partial \rho \bar{\omega}}{\partial t} = A_{in} P_{in} - A_{out} P_{out} - \frac{V \rho}{L} \left(f \frac{L}{D} + k \right) \frac{|\bar{\omega}| \bar{\omega}}{2} - LA \rho \bar{g} \sin \theta, \quad (2.2.10)$$

where \bar{g} acceleration due to gravity,

θ angle with relation to the horizontal.

$\nabla \cdot [\tau]$ is replaced with the term containing the friction coefficient f and resistance k . If the body force is only gravity, $\rho \bar{f}$ is replaced by an elevation change term. If a pump is present, an extra term must be added to consider an external force acting through pump head, ΔP_{pump} .

If $A_{in} = A_{out} = A$ and interpreting the mass flow as $W = \rho \omega A$ and volume $V = AL$, (2.2.10) becomes:

$$\frac{\partial W}{\partial t} = \frac{A}{L} \left[(P_{in} - P_{out}) - \left(f \frac{L}{D} + k \right) \frac{|W|W}{2\rho A^2} - \rho g L \sin \theta \right] \quad (2.2.11)$$

which is the form typically used in this work.

Conservation of Energy:

Substituting

$\psi = \rho(e + \bar{\omega} \cdot \bar{\omega} / 2) \equiv$ the sum of internal and kinetic energies per unit volume,

$\Gamma = \rho \bar{f} \cdot \bar{\omega} + E \equiv$ the work done by body forces and heat generated per unit volume,

$\Pi = \bar{q} + [\sigma] \cdot \bar{\omega} \equiv$ the heat conduction across the surface and the work done by stress forces,

into (2.2.3), we have the integral form:

$$\iiint_V \frac{\partial \rho e}{\partial t} dV + \iint_S \rho e \bar{\omega} \cdot \bar{n} dS = - \iint_S \bar{q} \cdot \bar{n} dS + \iiint_V E dV + \iiint_V [\tau] \nabla \bar{\omega} dV - \iint_S P \bar{V} \cdot \bar{\omega} dV \quad (2.2.12)$$

where e internal energy,

\bar{q} surface heat flux, and

E internal heat sources and sinks.

This equation can be used to generate the heat conduction equation for solids. By setting fluid velocity to zero and converting surface integrals to volume integrals we get the distributed form:

$$\frac{\partial(\rho e)}{\partial t} = -\nabla \cdot \bar{q} + E \quad (2.2.13)$$

Using Fourier law for heat conduction: $\bar{q} = -k\nabla T$ and replacing $\frac{\partial e}{\partial t} = c_v \frac{\partial T}{\partial t}$, we have the classical form of the heat conduction equation:

$$\rho c_v \frac{\partial T}{\partial t} = -\nabla \cdot k\nabla T + E \quad (2.2.14)$$

where k heat conductance,
 c_v specific heat at constant volume.

If we use enthalpy rather than internal energy, defining enthalpy

$$h = e + \frac{P}{\rho} \quad \text{and} \quad H = \rho V h,$$

we can transform (2.2.12) into:

$$\frac{\partial H}{\partial t} = \sum W h + Q + \iiint_V [\tau] \nabla \bar{\omega} dV - \iiint_V \left[\frac{\partial P}{\partial t} + \bar{\omega} \cdot \nabla P \right] dV \quad (2.2.15)$$

where $Q = -\iint_S \bar{q} \cdot \bar{n} dS + \iiint_V E dV$ is the heat flux term plus generation term. The integral term involving pressure is often neglected since it is usually negligible compared to other terms. The turbulent heating term is usually approximated by adding a friction heat into Q or can be neglected.

Neglecting turbulent heating and pressure terms, we have

$$\frac{\partial H}{\partial t} = \sum W_{in} h_{in} - \sum W_{out} h_{out} + Q \quad (2.2.16)$$

where the indexes *in* and *out* correspond to inflow and outflow of mass and enthalpy into and out of the volume V .

2.2.2. Equation of State [5]

From the conservation equations, we have three equations (mass, momentum and energy) and four unknowns:

- 1) density ρ , or mass $M = \rho V$,
- 2) velocity ω , or mass flow W , or momentum $\rho\omega$
- 3) energy e , or enthalpy h , or temperature $T = \text{fn}(e)$ or $\text{fn}(h)$, and
- 4) pressure P .

The fourth equation required for closure is the equation of state:

$$P = \text{fn}(h, \rho) \quad \text{or} \quad \rho = \text{fn}(P, T), \quad \text{etc.}$$

From thermodynamics, the equation of state of a substance is the relationship between any four thermodynamic properties of the substance, three of which are independent. For example, the equation of state involves pressure P , volume V , temperature T and mass of the system M :

$$\pi(P, V, T, M) = 0$$

If any three of the four properties are fixed, the fourth is determined. The equation of state can also be written in a form of specific values of the properties:

$$\pi(P, v, T) = 0$$

where $v = V/M$, the specific volume. If any two of the specific properties are fixed, the third is determined.

The practical way to present fluid properties is to tabulate or to formulate each property as function of the independent parameters P and T . Given values of P and T , the calculation of other thermodynamic properties is straightforward:

$$h = h(P, T), \quad \rho = \rho(P, T), \quad \text{etc.}$$

However, determinations of P or T from other known properties are not direct since interpolation and iteration are required. Thus, since the numerical solution of the conservation equations yields mass (ρ) and energy (h) as a function of time, it is the mass and energy which are independent parameters:

$$P = P(\rho, h) \quad \text{and} \quad T = T(\rho, h).$$

The iterative method for determination of P and T from known ρ and h starts by guessing a pressure. Given $P = P_{\text{est}}$, we calculate enthalpies of saturation liquid and vapour phases $h_f(P)$ and $h_g(P)$. If $h < h_f$, the fluid is single phase liquid. If $h > h_g$, the fluid is single

phase vapour. For the single phase fluid, temperature is also guessed $T=T_{\text{est}}$. Then density and enthalpy can be found:

$$h^* = h(P, T) \quad \text{and} \quad \rho^* = \rho(P, T).$$

The next iteration pressure and temperature are

$$P = P_{\text{est}} + \frac{\Delta\rho}{\left(\frac{\partial\rho}{\partial P}\right)_h} + \frac{\Delta h}{\left(\frac{\partial h}{\partial P}\right)_\rho} \quad \text{and} \quad T = T_{\text{est}} + \frac{\Delta\rho}{\left(\frac{\partial\rho}{\partial T}\right)_h} + \frac{\Delta h}{\left(\frac{\partial h}{\partial T}\right)_\rho}$$

where $\Delta\rho = \rho^* - \rho$ and $\Delta h = h^* - h$.

For two-phase mixture $h_f \leq h \leq h_g$, temperature is saturation temperature $T_{\text{sat}}(P)$, and density is

$$\rho^* = \rho_f + x\rho_{fg}, \quad \text{with a quality } x = \frac{h - h_f}{h_g - h_f}.$$

The next iteration pressure can be

$$P = P_{\text{est}} + \frac{\Delta\rho}{\left(\frac{\partial\rho}{\partial P}\right)_{\text{sat}}}, \quad \text{where } \Delta\rho = \rho^* - \rho.$$

The iteration process is repeated until P and T have converged to some tolerance.

The rate method will eliminate the need for iteration with no loss in accuracy. We wish to relate rates of change in pressure to rates of change in ρ and h :

$$dP = G_1 d\rho + G_2 dh \quad \text{or}$$

$$\frac{dP}{dt} = G_1 \frac{d\rho}{dt} + G_2 \frac{dh}{dt} \quad (2.2.17)$$

since $\frac{d\rho}{dt}$ and $\frac{dh}{dt}$ (or equivalently, $\frac{dM}{dt}$ and $\frac{dH}{dt}$) are available from mass and energy

conservation equations. That is:

$$h = h(P, T) \quad \Rightarrow \quad dh = \left(\frac{\partial h}{\partial P}\right)_T dP + \left(\frac{\partial h}{\partial T}\right)_P dT$$

$$\rho = \rho(P, T) \quad \Rightarrow \quad d\rho = \left(\frac{\partial\rho}{\partial P}\right)_T dP + \left(\frac{\partial\rho}{\partial T}\right)_P dT$$

$$\therefore dP = \frac{\left(\frac{\partial\rho}{\partial T}\right)_P}{(\text{DENOM})} dh - \frac{\left(\frac{\partial h}{\partial T}\right)_P}{(\text{DENOM})} d\rho, \quad \text{and}$$

$$dT = \frac{\left(\frac{\partial \rho}{\partial P}\right)_T}{(\text{DENOM})} dh - \frac{\left(\frac{\partial h}{\partial P}\right)_T}{(\text{DENOM})} d\rho \quad (2.2.18)$$

$$\text{where } (\text{DENOM}) = \left(\frac{\partial h}{\partial P}\right)_T \left(\frac{\partial \rho}{\partial T}\right)_P - \left(\frac{\partial h}{\partial T}\right)_P \left(\frac{\partial \rho}{\partial P}\right)_T.$$

G_1 and G_2 in (2.2.17) are functions that depend only on the local saturation fluid properties and their slopes at the local pressure. Their values are given in Table 2.1.

Equation (2.2.18) can be cast in term of the extensive form by noting $\rho = V/M$ and $h = H/M$. Thus, substituting:

$$\frac{d\rho}{dt} = \frac{1}{V} \frac{dM}{dt} - \frac{M}{V^2} \frac{dV}{dt} \quad \text{and} \quad \frac{dh}{dt} = \frac{1}{M} \frac{dH}{dt} - \frac{H}{M^2} \frac{dM}{dt}$$

into (2.2.18), after some simplification and rearrangement we have:

$$\frac{dP}{dt} = \frac{F_{1P} \frac{dM}{dt} + F_{2P} \frac{dH}{dt} + F_{3P} \frac{dV}{dt}}{F_{4P} M_g + F_{5P} M_f} \quad (2.2.19)$$

Similarly,

$$\frac{dT}{dt} = \frac{F_{1T} \frac{dM}{dt} + F_{2T} \frac{dH}{dt} + F_{3T} \frac{dV}{dt}}{F_{4T} M_g + F_{5T} M_f} \quad (2.2.20)$$

where M_g mass of vapour phase, $M_g = xM$

M_f mass of liquid phase, $M_f = (1-x)M$.

The values of functions F_{iP} and F_{iT} are given in Table 2.2.

The rate method has some advantages compared with the iterative method. First, the equation set (mass, flow, enthalpy and pressure) consists of four equations, characterizing the four main actors: M , W , H and P . Second, it permits the numerical calculation of pressure without iteration. Since the pressure can be explicitly expressed in terms of slowly varying parameters, an implicit scheme can be formulated and coded.

Table 2.1. G functions for the rate form of equation of state

Case	G_1	G_2
Two phase mixture (all derivatives along saturation line)	$\frac{h_{fg} v^2}{v_{fg} \left(\frac{\partial h_f}{\partial P} + x \frac{\partial h_{fg}}{\partial P} \right) - h_{fg} \left(\frac{\partial v_f}{\partial P} + x \frac{\partial v_{fg}}{\partial P} \right)}$	$\frac{v_{fg}}{v_{fg} \left(\frac{\partial h_f}{\partial P} + x \frac{\partial h_{fg}}{\partial P} \right) - h_{fg} \left(\frac{\partial v_f}{\partial P} + x \frac{\partial v_{fg}}{\partial P} \right)}$
Single phase for pressure (P)	$\frac{\left(\frac{\partial h}{\partial T} \right)_P}{\left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial h}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial T} \right)_P \left(\frac{\partial h}{\partial P} \right)_T}$	$\frac{-\left(\frac{\partial \rho}{\partial T} \right)_P}{\left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial h}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial T} \right)_P \left(\frac{\partial h}{\partial P} \right)_T}$
Single phase for temperature (T)	$\frac{-\left(\frac{\partial h}{\partial P} \right)_T}{\left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial h}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial T} \right)_P \left(\frac{\partial h}{\partial P} \right)_T}$	$\frac{\left(\frac{\partial \rho}{\partial P} \right)_T}{\left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial h}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial T} \right)_P \left(\frac{\partial h}{\partial P} \right)_T}$

Table 2.2. F functions for the rate form of equation of state

Case	F ₁	F ₂	F ₃	F ₄	F ₅
Two phase mixture (all derivatives along saturation line)	$h_g v_f - h_f v_g$	v_{fg}	$-h_{fg}$	$v_{fg} \frac{\partial h_g}{\partial P} - h_{fg} \frac{\partial v_g}{\partial P}$	$v_{fg} \frac{\partial h_f}{\partial P} - h_{fg} \frac{\partial v_f}{\partial P}$
Single phase for pressure (P)	$\rho \left(\frac{\partial h}{\partial T} \right)_P - h \left(\frac{\partial \rho}{\partial T} \right)_P$	$-\left(\frac{\partial \rho}{\partial T} \right)_P$	$-\rho^2 \left(\frac{\partial h}{\partial T} \right)_P$	<i>subcooled:</i> 0 <i>superheated:</i> $\left(\frac{\partial h}{\partial P} \right)_T \left(\frac{\partial \rho}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial \rho}{\partial P} \right)_T$	<i>subcooled:</i> $\left(\frac{\partial h}{\partial P} \right)_T \left(\frac{\partial \rho}{\partial T} \right)_P - \left(\frac{\partial \rho}{\partial P} \right)_T \left(\frac{\partial \rho}{\partial P} \right)_T$ <i>superheated:</i> 0
Single phase for temperature (T)	$\rho \left(\frac{\partial h}{\partial P} \right)_T - h \left(\frac{\partial \rho}{\partial P} \right)_T$	$-\left(\frac{\partial \rho}{\partial P} \right)_T$	$-\rho^2 \left(\frac{\partial h}{\partial P} \right)_T$	$-F_{4P}$	$-F_{5P}$

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2.2.3. Heat Generation and Heat Transfer in Nuclear Reactors [6]

In nuclear reactors, fission is the origin of thermal energy generation. The thermal energy released by fission in the fuel elements is transferred by heat conduction to the surface of the fuel elements and then by convection to the coolant which circulates around the fuel.

Heat Generation:

Approximately 200 MeV of energy emitted during a fission event is converted to heat, most of which ($\approx 94\%$) takes place within the fuel due to fission fragments and very short-range β -rays. The energy released during neutron thermalization takes place in the moderator ($\approx 5\%$). In thermal reactors fission is mostly induced by thermal neutrons. The volumetric heat generation in the reactor fuel is given by:

$$q_v(\mathbf{r}, t) = \alpha E_R \Sigma_f \Phi(\mathbf{r}, t) \quad (2.2.21)$$

where α the portion of thermal energy deposited in the fuel,
 E_R the recovered fission energy which is ≈ 200 MeV/fission,
 Σ_f fission macroscopic cross-section of fuel for thermal neutrons,
 Φ thermal neutron flux.

Usually, the fission power rather than the neutron flux is used in this thesis. Using the definitions as in (2.1.2) and (2.1.3) for thermal neutrons, we can rewrite (2.2.21) as:

$$q_v(\mathbf{r}, t) = q_{v0} \alpha p(t) \psi(\mathbf{r}) \quad (2.2.22)$$

where q_{v0} is the average power density at normalized reactor power ($p = 1$). That is the thermal energy released per unit volume of the fuel.

Equation (2.2.22) shows that to determine the distribution of heat generation rate in the reactor core, we need to know the shape of the neutron flux throughout the regions of the core which contains fuel. The accurate determination of neutron flux or power density distribution throughout the core is a complicated process and is beyond the scope of this work. For heterogeneous cylindrical reactors, the axial flux shape is usually given by a function of axial flux shape, $\varphi(z)$, while the radial flux is discretized by the fuel channel factor, k_r , which is the ratio of the power generated in the channel to the power generated in an average channel. Thus:

$$q_v(\mathbf{r}, t) = q_{v0} \alpha p(t) k_r \varphi(z) \quad (2.2.23)$$

k_r and $\varphi(z)$ are determined by neutron flux measurements in the reactor core.

In addition to the fission heat, there is another heat source which must be taken into account. That is the heat released by the decay of fission products within the fuel. For research reactors, reactor power is assumed to be the neutron power and not the thermal power, therefore, the more accurate form of (2.2.23) must be

$$q_v(\mathbf{r}, t) = q_{v0} [\alpha p(t) + p_{fp}(t)] k_r \varphi(z) \quad (2.2.24)$$

where $p_{fp}(t)$ is fission product decay heat contribution to the thermal power. The amount of decay heat depends on the inventory of fission products at time t . The build-up of fission products can be approximated by

$$p_{fp}(t) = \sum_{i=1}^N \lambda_i \bar{\omega}_i(t)$$

$$\frac{\partial \bar{\omega}_i}{\partial t} = \gamma_i p(t) - \lambda_i \bar{\omega}_i(t) \quad (2.2.25)$$

where $\bar{\omega}_i$ weighted inventory of i -th group of fission products,

γ_i fission yield,

λ_i decay constant,

N number of pseudo-fission product groups to model the decay heat source.

Heat Transfer:

The heat generated within fuel is first transferred by heat conduction to the fuel surface, across the cladding, and then by convection to the coolant flow. For heat convection, Newton law of cooling is used:

$$q_c = h(T_c - T_b) \quad (2.2.26)$$

where q_c local heat flux at interface between surface and fluid flow,

T_c surface temperature,

T_b average fluid bulk temperature,

h the heat transfer coefficient.

The convective heat transfer coefficient is greatly dependent on the flow regime and fluid properties. Usually, the heat transfer coefficient, h , is calculated through a dimensionless parameter, Nusselt number:

$$h = Nu \frac{k}{D} \quad (2.2.27)$$

where D hydraulic diameter = $4 \frac{(\text{flow area})}{(\text{wetted perimeter})}$,

k fluid thermal conductance taken at T_b ,

Nu Nusselt number.

The empirical correlation is actually for Nusselt number as function of other parameters such as Reynold number, fluid properties, geometry, etc.: $Nu = fn(Re, Pr, \dots)$.



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2.2.4. Empirical Correlations

Friction Coefficient:

Darcy formula for friction pressure drop in ducts is:

$$\Delta P_f = \left(f \frac{L}{D} + k\right) \frac{W^2}{2\rho A^2}, \quad (2.2.28)$$

where f is the friction coefficient, calculated by an empirical correlation $f = f_n(\text{Re})$, and k is the fitting loss coefficient, usually tabulated for sudden changes of flow area or flow direction.

For flow in smooth channel, the friction coefficient f can be expressed as [12]:

$$f = 4 \left(a + \frac{b}{\text{Re}^m} \right), \quad (2.2.29)$$

where a , b and m depend on a dimensionless parameter, Reynold number:

$$\text{Re} = \frac{WD}{A\mu},$$

where μ is the dynamic viscosity of fluid.

$\text{Re} \leq 2100$	$a=0$	$b = 16$	and	$m = 1.$
$2100 < \text{Re} \leq 4000$	$a=0.0054$	$b = 2.3 \times 10^{-8}$	and	$m = -2/3$
$\text{Re} > 4000$	$a=1.28 \times 10^{-3}$	$b = 0.1143$	and	$m = 3.2154$

To account for the temperature dependence of fluid properties, the friction coefficient f of (2.2.27) should be multiplied by a factor

$$k_\mu = \left(\frac{\mu_b}{\mu_w} \right)^m$$

where μ_b and μ_w are fluid viscosity taken at bulk and wall temperatures respectively. According to Deissler, $m=-0.58$ for heating and $m=-0.5$ for cooling. Also, to account for the configuration of the flow channel, the friction coefficient should be multiplied by another factor varying from $k_c = 1$ for circular tube to $k_c = 1.5$ for infinite parallel plate channel.

For some simple cases, the fitting coefficient k can be estimated as follows:[2]

sudden expansion in flow area: $k = 2 \left(\frac{A_2}{A_1} - 1 \right), \quad A_2 > A_1;$

sudden contraction in flow area: $k = 1.4 \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right], \quad A_1 > A_2;$

change in flow direction: $k = 0.6$ for 90° bend.

Heat Transfer Coefficient:

The heat transfer coefficient is actually a complicated function of the flow conditions, thermodynamic properties of the fluid, geometry and dimension of the surface. Specific correlations exist for the full range from single phase laminar flow to two phase turbulent flow. For laminar flow in circular tube, the analytical values of Nu are found to be $Nu = 4.36$ with constant heat flux and $Nu = 3.96$ with constant wall temperature.

For laminar flow, $Re < 2300$, Sieder-Tate correlation can be used

$$Nu = 1.86 \left(Re Pr \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \quad (2.2.30)$$

where $Pr = \frac{k}{c_p \mu}$ is Prandtl number. μ_b and μ_w are taken at mean fluid temperature and at wall surface temperature respectively. This correlation is valid if

$$\left(Re Pr \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \geq 2 \quad (\text{Whitaker condition}).$$

For transition and turbulent flow, $2300 \leq Re < 5.10^6$, Gnielinski correlation is the best choice:

$$Nu = \frac{(f/2)(Re - 1000) Pr}{1 + 12.7\sqrt{f/2}(Pr^{3/2} - 1)} \quad (2.2.31)$$

where $f = (1.58 \ln Re - 3.28)^{-2}$. To account the temperature-dependent viscosity, a factor of $\left(\frac{\mu_b}{\mu_w} \right)^{0.14}$ is multiplied as recommended by Deissler. Also, to account the configuration of flow channel, Nusselt number is multiplied by a channel factor k_c . For example, $k_c = 0.785$ for square channel, $k_c = 1$ for circular tube, up to $k_c = 1.89$ for infinite parallel plate channel [13].

Onset of Nucleate Boiling:

In research reactors using plate-type fuel elements, the bubble formation in heated channels can lead to critical phenomena. The onset of nucleate boiling (ONB) is then considered as the first warning, and the heat flux that initiates ONB is frequently used as a thermal design constraint. Actually, the ONB is taken as a safety limit in steady state

conditions although it does not correspond to any critical event. For the first bubbles attached to the wall appearing, it must be under the conditions that the wall temperature must exceed the coolant saturation temperature T_{sat} in a $\Delta T_{sat} = T_c - T_{sat}$. This is normally referred to as 'wall overheating'.

The ONB heat flux can be estimated by Bergles-Rosenow correlation:

$$q_{ONB} = 15.6 P^{1.156} (\Delta T_{sat})^n, \quad n = 2.3 P^{-0.0234} \quad (2.2.32)$$

where q_{ONB} ONB heat flux, [Btu/ft²-hr],

P pressure, [psia],

ΔT_{sat} wall overheating, [°F]: $\Delta T_{sat} = T_{cONB} - T_{sat}$.

The margin to ONB is defined as the ratio of the ONB heat flux to the local heat flux at the same point:

$$\text{Margin to ONB} = \frac{q_{ONB}}{q_c} \quad (2.2.33)$$

Critical Heat Flux:

During an accidental situation, the phenomenon known as departure from nucleate boiling (DNB), can appear. As a consequence of very high heat fluxes, large production of bubbles close to the surface takes place, an insulating vapour layer is formed which prevents the normal heat exchange between the wall and the coolant. The wall temperature increases drastically due to this phenomenon and may lead to the melting or deformation of the wall.

To calculate the margin to the DNB, the critical heat flux (CHF) is estimated. The CHF is the upper bound for design purposes and can be given by Mishak's correlation:

$$q_{CHF} = 1.51 \times 10^6 (1 + 0.1198\omega)(1 + 0.00914\Delta T_{sub})(1 + 0.19P), \quad (2.2.34)$$

where q_{CHF} critical heat flux, [W/m²]

ω flow velocity, [m/s]

P pressure, [bar]

$\Delta T_{sub} = T_{sat} - T_b$, [°C].

The margin to DNB is defined as the ratio of the CHF to the local heat flux at the same point:

$$\text{Margin to DNB} = \frac{q_{CHF}}{q_c} \quad (2.3.35)$$