

Chapter IV

Conclusions & Further work

We have been successful in our attempt to define a logic with types that preserves most of the basic results from traditional predicate logic. We were able to define a syntax that is quite similar to the traditional one. As for the semantics, it looks more complicated but in truth the only significant difference is in the universe. The interpretations of terms and formulas are similar to the old ones. The definition of formal proofs is also similar, and we can show the power of this logic by proving some basic theorems in group theory which cannot be done in traditional logic. Many metatheorems in traditional logic are also preserved. In particular, the Soundness theorem still holds. But in the end it turns out that, this system is not complete.

As we expected, we found a counterexample to the Compactness and Completeness theorems. We discovered this counterexample after we tried to construct the ultraproduct for this logic. We were almost successful in constructing the ultraproduct when we found that if we allowed a variable to have a function type (i.e., if we allowed function variables), the counterexample which we have shown would occur.

We hope that if we restrict the variables to have types of order one, we will be able to construct an ultraproduct. In that case we would be able to prove the Compactness theorem.

Even though our logic is not complete and seems to be more complicated, it does offer more expressive power than traditional predicate logic. And if our expectation concerning the ultraproduct is true, we will have a new logic that satisfies the Compactness theorem and still has higher expressive power than first-order predicate logic.